Asia-Pacific
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MATHEMATICAL MODELING COURSE IN MATHEMATICS CURRICULUM : SOME BEST PRACTICES IN APEC ECONOMIES

Human Resources Development Working Group

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FOREWORD

Having worked in math education, there is a concern about a frequently-asked question about the role of Mathematics in the student's future for winning the job seeking competition with other graduates with other backgrounds. Some of them are not worry about the answer because they will be like the teachers and lecturers, who teach and do research on Mathematics. However, a big part of them should be given a good answer. We think the good way to giving the answer is make them experience Mathematical Modeling in solving real-world problems. By organizing workshop on "Promoting Best Practices on Mathematical Modeling Course in Mathematics curriculum of APEC economies", we want to enhance understanding on the importance of designing Mathematical Modeling course within the curriculum which have direct link to real-world-driven problems. This book contains the results of this workshop.

We would like to show our gratitude to APEC (Asia Pacific Economic Cooperation) who give funding through the projet S HRD 06 11A with Project Overseer Prof. Dr. Edy Soewono, and also to Center of Mathematical Modeling Research and Simulation (P2MS-ITB) with Chair Dr. Nuning Nuraini, who help us to organise this workshop. We are also grateful for the support from Faculty of Mathematics and Natural Science with Dean Prof. Dr.rer.nat Umar Fauzi, and from Research and Development Division, Indonesia Ministry of Education and Culture with Head of Division Prof. Dr. Khairil Anwar Notodiputro.

Hopefully this book is worthwhile to read as the beginning step to promote the development of Mathematical Modeling course worldwide, especially in APEC economies.

Novriana Sumarti, Ph.D.

Editor

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I. APEC Workshop on Mathematical Modeling Course at ITB

Novriana Sumarti
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I.1 The objectives of the workshop

One of the main concerns being faced by Mathematics educators at all levels is the inability of the instructors to motivate students for appreciating as well as understanding Mathematics. This is due to the fact that most Mathematics educators are lacking of problem solving skill in real world problems and Mathematics are being thought with no neccesity to relate to their relevances to real world problems. This contributes to demotivating students in Math and less competitiveness of higher education graduates major in Mathematics at the job markets. On the other hand, the role and demand of Mathematics, especially Mathematical modeling and simulation, in shaping up the fast growing technological development is very strategic and unavoidable in understanding complex phenomena.

A course in Mathematical Modeling is essentially needed to bring the real world problems into class room activity so the student get experience in direct-contact with the real world. The skill and competence required by stakeholders such as learning to learn, problem solving and creative thinking, interpersonal skills, teamwork and negotiation, and leadership can be well simulated and learned during the modeling activities.

In general, the main difficulty in delivering Mathematical modeling course is due to the long time existing paradigm in Mathematics communities that mastering a lot of Mathematical concepts and methods is enough to be good mathematician. As a consequence, attitude in appreciating and in dealing with real world problems (an important soft required in the job market) is not developed. The current irrelevance of Mathematics education to the real world problems affects the graduates have limited skills to adapt to new chalanges so they will end up mostly only with traditional career choices, such as teachers and researchers. It is necessary now to get the message through, using a media such as an industrial mathematician forum, especially among young mathematicians.

This project aims to identify ways in which Mathematical Modeling curriculum are being defined, developed, and practiced in the APEC region. A flexible course about Mathematical Modeling skills in the Mathematics undergraduate curriculum, which is adaptable to local conditions, could prepare the students to be adaptable and professional workforce and improve their skill and competencies in almost all fields of works. In sequence their career options could be broader, that are any work places that have research and development units, such as oil companies, banking industries, insurance companies, airline companies and others.

This project will put into practice the Yokohama Declaration in 2010 that will implement policies to enhance education and training with equal opportunities for women, youth, and elderly. The project will also promote the Inclusive Growth element of the APEC Leaders' Growth Strategy by improving the quality of education, especially in Mathematical Modeling of real world.

The project will support the Inclusive Growth attribute of the APEC Leaders' Growth Strategy, especially in promoting job creation, human resource development, and active labour market policies by improving the quality of education focused on skills and competencies needed in the 21st century workplace.

The project will also help to implement the Action Plan for Developing Human Resources, Vigorously Promoting Employment, and Achieving Inclusive Growth developing which includes "Promote competencies and skills of future work through quality technical and vocational education and training to enhance employment and build a more competent APEC workforce".

This project could be included in the EDNET project that supports priority areas set by the APEC Education Ministers at the 4th AEMM in June 2008 in Lima, Peru, that are "Mathematics and Science Education", where the skills are essential to navigating the data-driven and technological world of the 21st century, and "Career and Technical Education (CTE)", which covers academic education combined with real-world application and the workforce with Mathematics Modeling skills is able to adapt with new technological innovations and workers transition to multiple jobs over a lifetime.

I.2 Speakers and Participants

This document is the results of a workshop on "Promoting best practices on Mathematical Modeling Course in APEC economies", in October 22-23, 2012, at Institut Teknologi Bandung, Indonesia. The participants are from Malaysia, Thailand and Indonesia. Figure I.1 shows the cities of origin of the participants which are spead from Banda Aceh (5° 31' North and 95° 25' East) which is on the left end of map, to Jayapura Papua (2° 32' South and 40° 42' East) which is on the right end of the map. Some of participants are also coming for ALCoB (APEC Learning Community Builder) Indonesia section.

Figure I.1: Cities of origin of the participants. Picture source: maps.google.com

Speakers were coming from Australia, Chile, Japan, USA, Indonesia, and UK. In the first day, the speakers are Dr. Jeffery Waldock (Sheffield Hallam University), who discussed "Developing Graduate Skills through Mathematical Modeling in the Higher Education Curriculum" (see chapter 2), Prof. Edy Soewono (ITB), who discussed "Mathematical Modeling at MA ITB : Bringing real world problems into class room activities" (see chapter 3), Mr. Joshua P. Abrams (Meridian Academy, USA), who discussed "Mathematical Modeling for High School Students" (see chapter 4), Mr. Roberto Araya (Center for Advanced Research on Education, Universidad de Chile), who discussed "Introducing Mathematical Modeling Skills in the Curriculum" (see chapter 5), and Prof. Masami Isoda (Center for Research on International Cooperation in Educational Development, University of Tsukuba, Japan) who discussed "Mathematical Modeling for Emergency Preparedness Education".

I.3 Presentation from Prof. Masami Isoda

In the presentation of Prof. Masami Isoda, the discussion is about "What we can do for teaching mathematical modeling within the school mathematics". In the Japanese university curriculum, there is no course for mathematical modeling in the pure math departments. When students are employed, each company teaches mathematical modeling based on their necessity. In the other departments in engineering and so on, mathematical models are taught as a part of their subjects, not the part of math. For getting teachers certifications, students must take the course for mathematics education. Most of students may have a chance to study what it is because it is enhanced school curriculum as a part of mathematical activities.

Mathematical model and modeling are key ideas for mathematical science however they are not key activity for mathematicians. Mathematical Modeling is also not key activity for scientists. Mathematical Model usually uses algebraic representations and not so many elementary geometry. Why it is necessary for mathematics education because Modeling is for developing the ability for problem solving in real world. Its teaching how to use math in real world.

There is alienation between "theoretical modeling" and "experiential modeling" (Berry and Houston, 1995) Alienation between algebraically represented progressive models (the way of current modern mathematics) and geometrical represented Exact Science (historically existed and currently lost some parts). Mechanism under geometry does not need to take date because it is exactly correct.

Mathematical modeling is usually a partial activity of the group of scientists or inters subjective disciplinary activities. One example of this activity is in "Mathematical Modeling and Problem Posing on Earthquake and Tsunami: Scientific Research for the Disaster", example is shown in figure I.2, which is taken from http://math-info.criced.tsukuba.ac.jp/museum/dbook_site/Lectures-Samples-pub/files/EText.html. The presentation on this topic began with a sad story from Saori Endo, a first grade student at the Sendaiikuei High School, who lost her mother, grandmother and great-grand mother during the tsunami hit Japan in March 11th, 2011. The story can be found in http://math-info.criced.tsukuba.ac.jp/museum/dbook_site/READINGS-SAMPLES-pub/files/EText.html. It was devastating experience so it is important to have study of "Mathematical Modeling for Emergency Preparedness Education".

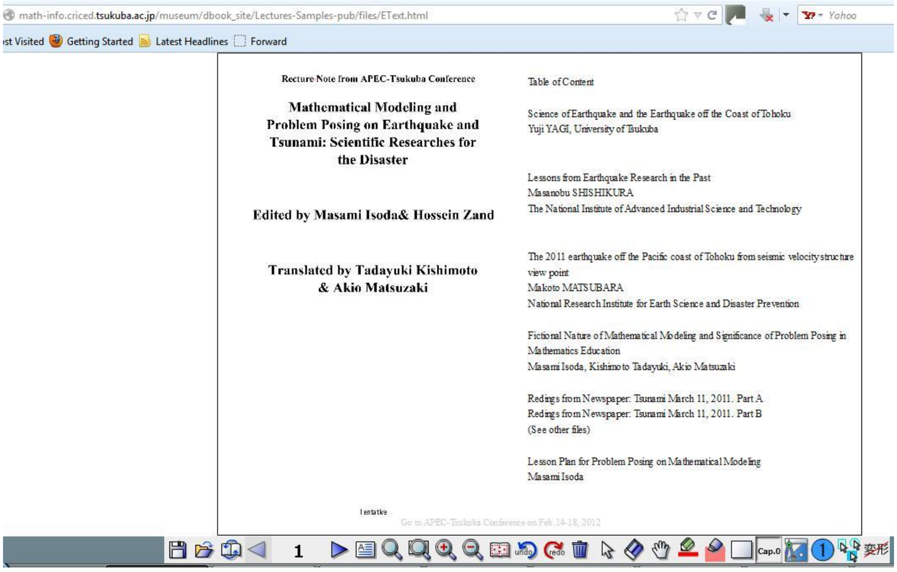

Figure I.2: Mathematical Modeling and Problem Posing on Earthquake and Tsunami

His other work for historical models in Mathematics can be found in http://math-info.criced.tsukuba.ac.jp/museum/ which an example is shown in Figure I.3.

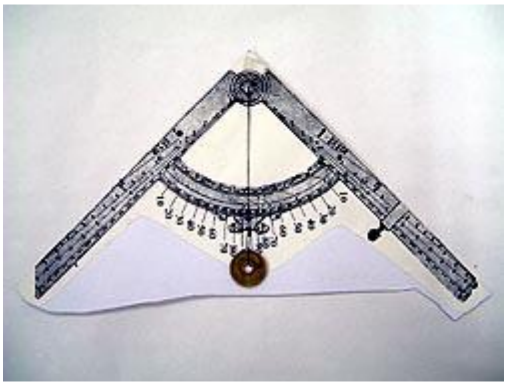

Figure I.3 : Military compass from history model of Mathematics

I.4 Presentation from Prof. Jonathan Borwein

The speakers in the second days are coming from the leaders of research centers which are multidiscipline and use mathematical modeling heavily in their research so fresh graduates of math have opportunity to work directly in these centers. First speaker is Prof. Jonathan Borwein from CARMA (Centre for Computer-Assisted Research Mathematics and its Applications) in University of Newcastle, Australia, which gives presentation via teleconference with title "CARMA and Me, for 23-10-2012 ITB-APEC Workshop".

Before starting, he mentioned that it is important to have a robust collaboration to develop research. Indonesia and Australia are potential to have the collaboration regarding the close geographic location so that the time difference is not too much. Firstly, he explained about Experimental Mathematics which is the core research of CARMA. Experimental mathematics is the use of a computer to run computations - sometimes no more than trial-and-error tests - to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search. One of the results of the experimental mathematics is Integer Relation Methods which put its algorithm, named PSLQ (Fergusson et.al. 1999), as one of top 10 algorithms of 20th century. PSLQ, which is based on partial sum of squares scheme implemented using QR decomposition, is the core to CARMA. There are many publications on Experimental Mathematics, which are including the books in Figure I.4.

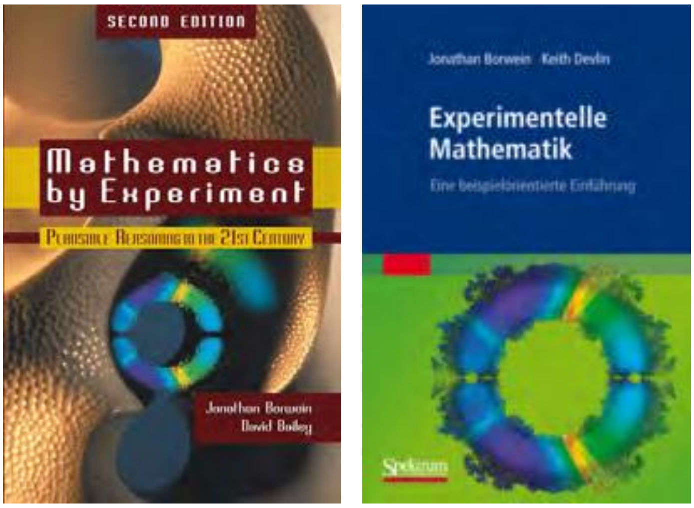

Figure I.4: Some of the Experimental Mathematics books

It is emphasized that Mathematics, as the language of high technology" which underpins all facets of modern life and current Information and Communication Technology (ICT), is ubiquitous. No other research centre exists focusing on the implications of developments in ICT, present and future, for the practice of research mathematics. CARMA fills this gap through exploitation and development of techniques and tools for computer-assisted discovery and disciplined data-mining including mathematical visualization.

The objectives of CARMA are:

- To perform Research and Development relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science) of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modeling of such activities. (C-OPT, NUOR and partners)
- To promote and advise on use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications. In 2010, University of Newcastle is the only Australian university who had received class ‘5’ in Applied Mathematics.

CARMA was founded in 2008 in University of Newcastle, however it has deep history relating to previous works of Prof Borwein and his brother (Peter Borwein), starting from 1982 at Dalhousie when they worked on fast computation. There were research centres founded from 1993 to 2012, which includes CECM (Centre for Experimental and Constructive

Mathematics), D-Drive (Dalhousie Distribute Research Institute and Virtual Environment), IRMACS (Interdisciplinary Research in the Mathematical and Computational Sciences), CARMA and C-OPT (Centre for Optimal Planning and Operations). In C-OPT, the optimization research has been conducted including "Using Mathematics to Maximize the Efficiency of Shared Infrastructure in Australia's Coal Export Supply Chain" (HVCCC, CSIRO, UoN) 2009-2012, and "Methods and Software for Efficiently Solving the Transportation Crewing Problem "(CTI, Monash, UoN) 2008-2012.

Roughly there are 40 current Members and Associates of CARMA from multidiscipline background, including Steering Committee (Assoc Directors for Applied/Pure/OR), External Advisory Committee (IBM, Melbourne, LBNL), Members and Students from Newcastle, and Associate Members from Everywhere.

Current research interests of Prof Borwein are including Optimization Theory and Applications, Nonlinear Functional Analysis, Computational Number Theory, and Algorithmic Complexity Theory. In the end of presentation, he explained one of topics in Computational Number Theory that is the computation of π number. Modern computation of π was settled a century ago. One motivation of this computation is the raw challenge of harnessing the stupendous power of modern computer systems. Substantial practical spin-offs accrue accelerating computations of π sped up the fast Fourier transform (FFT) - heavily used in science and engineering. It is also to bench-marking and proofing computers, since brittle algorithms make better tests.

I.5 Presentation from Prof. Septo R. Siregar

The second speaker is Prof. Septo R. Siregar from Faculty of Mining and Petroleum Engineering – ITB, and the title of his presentation is "The Role of Mathematical Modeling in Solving Oil & Gas Industries Field Problems". There is a fact that it is difficult to convince people in industry the importance of mathematics in industry. We need an interface between people industry and mathematicians which can be filled by people like him as an engineer which is more believed by the industry. He presented the works at OPPINET as the proof that mathematics is beneficial for oil and gas industry.

Oil and Gas Industries have a very large spectrum of problems, from the reservoir underground, the wellbore through which oil and gas are flowing to the surface, and then surface facilities. It is a big challenge to extract crude oil from the pores in the reservoir and then to bring it to the next surface facilities, such as transportation pipeline. In many cases the problems are quite complex and people in the fields are satisfied with engineering solutions. It is very convenient to find better solutions with the help of mathematical modeling. With mathematical modeling can improve and optimize the solution by more economical cost and it can go as far as we would like to go in modeling without any physical constraints. For example in this industry, if we do only 0.01% change in the pipeline plan, the cost is in millions dollars which is very expensive. With mathematical modeling, we can do optimization without spend very expensive cost because the model is in the papers and computers.

Prof Siregar is the chairs of two research consortium, which are OGRINDO (Oil and Gas Recovery for Indonesia) and OPPINET (Optimization in Pipeline Network). OGRINDO founded in 2004 is dealing with oil and gas within the reservoir in underground so we can improve the oil recovery by using the most suitable technology implemented in Indonesia. In figure I.5 it is shown the basic program of OGRINDO.

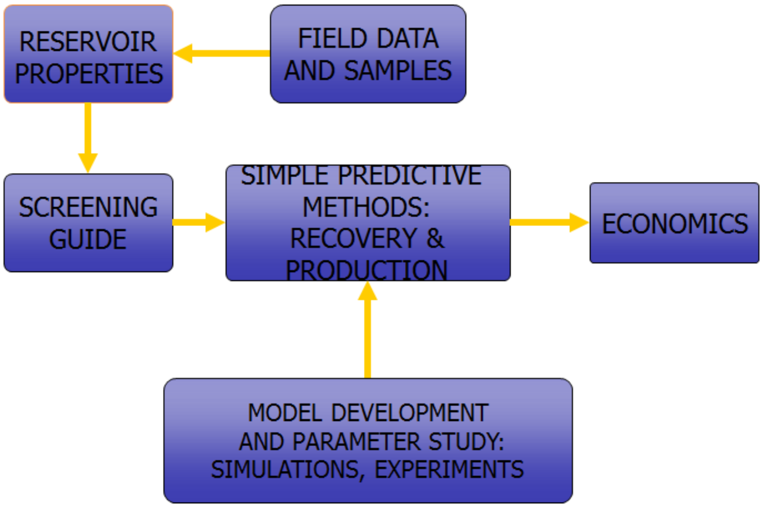

Figure I.5: the basic program of OGRINDO

There are about 30 permanent staffs in OGRINDO consisting experts from petroleum engineering, mathematics, biology, chemistry and physics. Many students from undergraduate and postgraduate programs are also involved in this activity. The companies supporting this research consortium are BOB PT. Bumi Siak Pusako, Pt. Chevron Pacific Indonesia, CNOOC SES Ltd, PT. Medco E & P Indonesia, PT. Pertamina E & P, PETROCHINA GROUP (JOB Pertamina-PetroChina Salawati, PetroChina-East Java, PetroChina International Bermuda, PetroChina International Jabung), and PT. Total E & P Indonesie.

The second research consortium is OPPINET which is founded in 2001. This consortium is aimed at overcoming oil and gas strategic technology in the near future, particularly in oil and gas transportation technology through mathematical modeling and software development. The inspiration to organize a consortium was coming from a paper by Daniel de Wolf (Université de Lille III, france) and Yves Smeers (Université Catholique de Louvain, Belgium) with title "Optimal dimensioning of pipe networks with application to gas transmission networks" which proposed a mathematical modeling in pipe network problems. We are also inspired by Prof. H. Neunzert ITWM from University of Kaiserslautern, Germany who was actively promoting the use mathematical modeling in solving real world problems.

The permanent researcher of OPPINET is about 13 people with multidiscipline backgrounds. There are also a quite many students and graduates who work in this consortium. Companies participants are including Chevron Pacific Indonesia, TOTAL E&P Indonesie, PT. Perusahaan Gas Negara (Persero), Tbk., POMA Companies (Badak NGL, Chevron Indonesia Company, PT. Pertamina Gas, TOTAL E&P Indonesie , VICO Indonesia), BOB PT. BSP – Pertamina Hulu and Badan Pengatur Hilir Minyak dan Gas Bumi.

Current big topics in the research of oil and gas industries problems being conducted are Well & Field Production Modeling, Oil Flow Modeling, Gas Flow Modeling, Multi Phase Flow Modeling, and Scrubber Modeling (POMA). One of results from the research is the making of software which is aimed to have better performance than related commercial software commonly used in oil and gas industries.

I.6 Sugestion

To conclude, the workshop was worthwhile in promoting best practices on mathematical modeling course in mathematics curriculum within APEC economies. Many participants, especially from Indonesia, have become aware of the importance of mathematical modeling course. In the next activity, there is a big hope that there will be more mathematics department in universities or schools who have course in mathematical modeling as good as it had been implemented currently. In the future, hopefully there will be large number of APEC economies who will be willing to participate in this kind of workshop. This workshop and the similar projects in the future aim to provide inputs and recommendations to the APEC Education Ministerial Meeting on the Mathematics and Science Education priority area. In figure I.6, it is shown the speakers and participants of this workshop.

Figure I.6: the speakers and participants of this workshop

II. Developing Graduate Skills through Mathematical Modeling in the Higher Education Curriculum

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II.1 Undergraduate Mathematics Programme

A fundamental question exists concerning what a Mathematics degree should contain - in particular, should it include ‘employability’ or 'graduate' skills? If so, to what extent does the current curriculum, together with the learning, teaching and assessment strategy that delivers, supports and assesses it, incorporate these principles already (and where it does, how successful is it?).

HE mathematics is often a direct extension of high school mathematics, tackling more complex topics in greater depth. For graduates to employable however, these core mathematical techniques, important though they are, need to be rooted in practice. A graduate should have some experience of representing the real world mathematically - making simplifying assumptions as necessary - deriving the mathematical expression of a problem from a contextual setting. The solution of the problem can be carried out through 'standard' analytical or numerical methods, but must then be interpreted in context for the benefit of a variety of audiences. This, in essence, is the modeling process - which involves many so-called 'graduate' skills, such as communication and problem solving. Further skills such as team working and leadership can be developed through the learning, teaching and assessment practice used in delivering these parts of the curriculum.

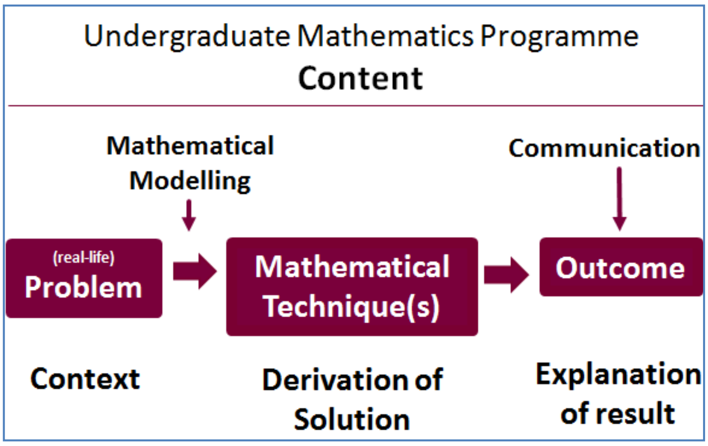

Figure II.1 : Undergraduate Mathematics Programme Content

Figure II.1 illustrates these ideas. A central part of mathematics programmes - at all levels - is the derivation of solutions to mathematically posed problems, as shown in the centre of the diagram. In some programmes of study a view is taken that the development of advanced techniques is of prime importance, and so this will form all or most of the curriculum. In others, attention is also given to the context in which the mathematical problem arises, and the simplifying assumption made when using mathematical modeling to represent the problem. For a graduate to be employable, it is also vital that they can interpret their results in context, taking account of assumptions made, and be able to communicate this in appropriate ways to different audiences. The inclusion in the curriculum of activities that require students to do this will help them develop a range of graduate skills.

II.2 Skills of graduates

Much has been said about the various generic skills that a graduate is expected to have developed. These include:

- Verbal communication
- Written communication
- Analysing and problem solving
- Team working and interpersonal skills
- Self awareness
- Personal planning, time management and organisation
- Initiative, enterprise
- Adaptability
- Numerical reasoning
- Information literacy and ICT skills

When identifying the curricular requirements of a particular mathematics programme it is perhaps helpful to ask the following questions:

- What skills do you think a graduate of your Mathematics programme **should** have?
- What skills do you think they **actually** have?
- What skills do you think they **need**?
- What is the **purpose** of a mathematics degree?

Skill Area	Expected to develop during course a lot or quite a lot	Expected to be very or quite important in future life
Logical thinking	98.2%	99%
Analytical approaches to working	96%	98%
Applying mathematics to real world problems	93%	95%
Thinking in abstract ways	91%	81%
Written communication	57%	86%
Oral communication	67%	92%
Making presentations	57%	82%

Table II.1 : Results from More Maths Grads project

Around five years ago in the UK, the **More Maths Grads** project was funded to address a shortage in the number of Mathematics graduates leaving University. One survey carried out as part of the project (Challis, N; Robinson, M and Thomlinson, M, 2009) asked 223 first year UG Mathematics students about their skills. 93% respondents rated the statement "I need a degree to get a good job" as 'very' or 'quite important'.

They were also asked about skills they expected to develop during their course, and which they expected to be important to them in their future life, whether in work or elsewhere. A selection of relevant results appears in Table 1.

The students were asked to distinguish between skills they thought were important and those they expected to develop as part of the course. It is quite noticeable that while some skills features highly on both lists - predominantly those such as logical thinking that would be expected of mathematics graduates - others, mainly the 'soft' skills such as communication, did not. Most students did not expect that their course would help them develop in these areas.

In the UK there is no prescribed content for degree courses. The UK Quality Assurance Agency (QAA) sets out the expectations that all providers of UK higher education are required to meet in so-called benchmark statements. The Mathematics Subject benchmark - a 'National Curriculum' for HE - says very little about mathematics (just that all courses should include calculus and linear algebra) but mentions many 'graduate' skills explicitly:

> *"MSOR (Mathematics, Statistics, and Operational Research) graduates will possess* ***general study skills****, particularly including the ability to learn independently"*
>
> *"They will also be able to* ***work independently*** *with patience and persistence, pursuing the solution of a problem to its conclusion. They will have had the opportunity to develop general skills of* ***time management*** *and* ***organisation****."*
>
> *"They will also have general* ***communication skills****, typically including the ability to* ***work in teams****, to* ***contribute to discussions****, to* ***write coherently*** *and to* ***communicate results clearly****."*

However, not all Mathematics degrees are, or need be, the same. When designing a curriculum, we need to decide whether we are interested in the career prospects of our graduates and, if so, in providing the right preparation for these careers. We must also remember that it is not just graduates entering the workplace that need this support - those planning a research career will also benefit from enhanced 'graduate' skills such as communication, organisation and reflection and action planning. Important questions we must ask include how to build this into the curriculum, what makes the biggest difference in helping our graduates into employment or further study and who is best placed to develop and deliver relevant sessions.

II.3 UK Student Surveys

As a discipline, mathematical sciences fares poorly in student surveys of skill development. A National Student Survey is carried out in the UK of all final year undergraduates each spring. These students are asked a range of questions about their experiences of their university, their course and their skills. Data from the survey for 2008, 2009, 2010 and 2011 are summarised at https://maths.shu.ac.uk/NSS/Skills2.php, showing the the ranked position of Mathematical Sciences against 41 other disciplines for questions around personal development. These are averaged across all institutions reporting (shown in brackets below each year) and presented in Table 2.

National Student Survey	**2008** (52)	**2009** (62)	**2010** (63)	**2011** (63)
Q19: The course has helped me present myself with confidence	66% 42nd	66% 42nd	70% 42nd	71% 42nd
Q20: My communication skills have improved.	66% 42nd	65% 42nd	70% 42nd	70% 42nd
Q21: As a result of the course, I feel confident in tackling unfamiliar problems.	75% 30th	77% 28th	77% 28th	79% 23rd
Q22: **Overall, I am satisfied with the quality of the course**	89% 4th	88% 3rd	89% 2nd	88% 8th

Table II.2: Results from National Student Survey for Mathematical Sciences

For the first two of these three questions, mathematical sciences ranks lowest of all disciplines – 42 out of 42. For the third, students are asked whether they feel that – as a result of the course – they are more confident in tackling unfamiliar problems. This is one that we would have expected mathematics students to do well in, and although the discipline is ranked more highly, it is still between 23rd and 30th out of the 42 disciplines reporting. It must be remembered, of course, that these ratings are of students' own perceptions, and it may be the case that they did not feel confident compared to their own expectations in talking problems.

The final question asks students whether, overall, they are satisfied with their course. It is noticeable that the results of this question do not align with an average of responses to all other questions – clearly, there are some areas that students feel are more important than others. It is striking that for the overall satisfaction – despite performing poorly in the personal development questions – mathematical sciences as a discipline performs very well, ranging between 2nd and 8th of the 42 disciplines represented. One conclusion that could be drawn from this is that students are not very worried about personal development, rating their course highly despite a poor showing in that area.

II. 4 Developing Graduate Skills in Mathematics Programmes

In UK, a mini-project commissioned by the National HE STEM Programme Mathematical Sciences Strand was conducted from April 2010 to March 2011. Three principal aims of this project are

1. To capture examples of what is currently being done within Mathematics programmes in UK HEIs to address the development of graduate skills,

2. To provide an appraisal of what approaches appear to have been successful in developing these skills, and
3. To use this to make recommendations for the further development of these and other programmes of study that wish to encourage the development of graduate skills.

Results of this project are including "Developing Graduate Skills in Mathematics Programmes‟ (March 2011), "Further work developing graduate skills in HE Mathematics Programmes" (June 2012), "Employer Engagement in Undergraduate Mathematics" (July 2012).

"Developing Graduate Skills in Mathematics Programmes‟ presents a series of short case studies providing examples of ways in which these skills have been successfully developed through curricular initiatives. Skills categorisation reflecting the key areas identified as "graduate skills" applies the following groupings:

1. Work-based learning and/or work-related learning.
2. Reflection and action-planning, including Personal Development Planning (PDP) and works portfolios.
3. Career Management Skills (CMS).
4. Employability skills (those not covered above, such as skills in team working, communication, leadership, autonomy and self-awareness).

Table II.3 provides a summary of these skills groups, as addresses by the set of case studies presented in the report.

Case Study		**Skill Group**			
		1	**2**	**3**	**4**
1:	Calculating Careers			Y	
2:	Personal Development Planning		Y		
3:	Professional and Graduate Skill Modules		Y	Y	Y
4:	**Integrative Use of Group Projects**				**Y**
5:	Venture Matrix	Y			Y
6:	Peer Assisted Learning				Y
7:	Progress Files		Y		Y
8:	LTA Approaches to Employability		Y		Y
9:	Placement Preparation			Y	
10:	**Art Gallery Problems**				**Y**
11:	Graduate Skills Development Programme (GDP)		Y	Y	Y
12:	Writing and Thinking				Y
13:	**Developing Models Applied to Business and Industry**	**Y**	**Y**		**Y**
14:	Case Study Booklets on Basic Mathematics				Y
15:	Every Student Counts				Y

16:	**Business Applications of Mathematics**	**Y**			**Y**
17:	Embedding Careers Awareness			Y	Y

Table II.3: Set of case studies and their developed skills.

Nearly all – 14 of 17 – address generic employability skills, particularly communication and team working, reflecting the importance attached to these skills, and if allowance is made for the fact that career planning and PDP include elements such as self-awareness, the coverage is 100%. The 4 cases designating Mathematical Modeling problems, highlighted above, also address these generic emplyability skills.

Points to consider when introducing graduate employability skills into the curriculum:

- Should a separate 'skills' module be included, or is it better to embed employability into the course structure? Research suggests the latter approach is much more successful, as students are more likely to engage with activities that are explicitly related to their programme of study.
- Explain to students the big picture - engaging them as partners in learning – and helping them to understand the purpose behind the design of the learning, teaching and assessment (LTA) activities that they are being asked to take part in. Help them recognise how each is designed to help develop a range of skills, leading to the full skill set expected of a graduate of their programme.
- Use LTA - particularly assessment - practice to help encourage the development of employability skills alongside technical skills. This approach will help staff recognise that curricular content need not be sacrificed for graduate skill development
- Ask students what they think they need to do to become better students. It is vital that students develop skills in self-awareness in order to be able to identify where they need to improve and to develop plans of action for doing so. Perhaps some type of learning log, regularly reviewed by staff, could be included in the programme.
- Should skills be assessed? If so, what part does this play in the overall assessment on the course? Assessment drives learning and engages and motivates students; even very small elements of assessment can help.
- External accreditation of skills (including extra-curricular skills)? If appropriate and possible, can some external agency – such as a professional body – provide further validation of the programme, underlining the importance of the skills developed through taking part in it.

The BSc Mathematics programme at Sheffield Hallam University is

- practical, applicable, mathematics
- extensive use made of technology
- skill development paramount
- 'threads' of activity through all years of the course:
  - 'core' mathematical techniques
  - modeling
  - technology
  - skills

Table II.4 shows the curriculum design, highlighting the thread of modeling activity that run through all levels of study.

Modelling	Core Maths	Statistics	Technology
Year 1 (Level 4)			
Mathematical Modelling	Number and Structure Mathematical Methods Graphs and Networks History of Mathematics	Statistics and Probability	Mathematical Technology Basic Computer Programming Dynamic Geometry
Year 2 (Level 5)			
Modelling 2	Linear and Discrete Mathematics Mathematical Analysis Dynamical Systems and Fourier Analysis Introduction to Knot Theory, Chaos and Fractals	Statistical Methods	Mathematics for Excel Mathematical Programming for the Web
Year 3: Industrial Placement (optional)			
Year 3/4 (Level 6)			
Advanced Modelling Case Studies Project	Modelling with Partial Differential Equations Fluid Flow Tensors Digital Signal Processing Abstract Algebra	Data Mining with Business Applications Statistics for Business	

Table II.4: Mathematical Modeling courses in the curriculum

II. 5 Examples of Modeling Activities used in the curriculum

Example 1: Mathematics of Digital Images

Students are familiar with digital images – and with post-processing them. Almost all digital photo post-processing uses Mathematics such as in reflection, rotation, changing brightness, changing contrast, tinting (altering colour balance), blurring, sharpening, sharpening, re-sizing, edge-detection, skew-strecth-shear, addition of noise, distortions, pattern detection, image arithmetics, morphing between images, and so on.

Digital images are composed of an array of pixels, each composed of separate red, green and blue components. Each of these components can take values between 0 and 255 which is normally expressed as hexadecimal values – in the form #RRGGBB. So red, for example, is #FF0000. Most common format for digital photographs is JPG, or JPEG.

An Excel add-in has been created which can:

- Import JPG images, placing the red, green and blue components as integers in separate worksheets.
- Resize images to fit the number of columns available.
- Recompile JPG images from the values in the R,G and B worksheets

For students this requires a preliminary introduction to binary and hexadecimal arithmetic. See figure II.2 for a screenshot of a program written to demonstrate the transformation of decimal number to binary, octal and hexadecimal, and it's representation as a greyscale. Figure II.3 is a screenshot of a program written to demonstrate the functional relationship between the input and output pixel values that is needed to change the brightness and contrast of an image.

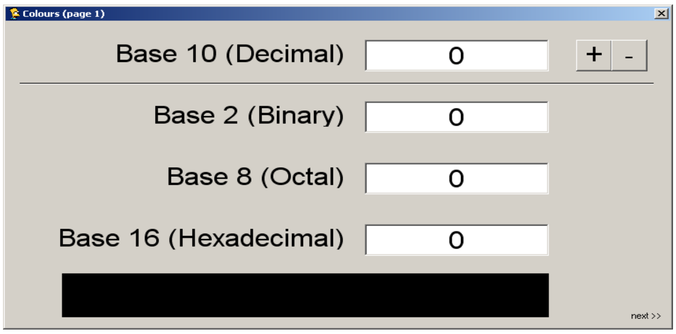

Figure II.2: Hexadecimal values of colour components

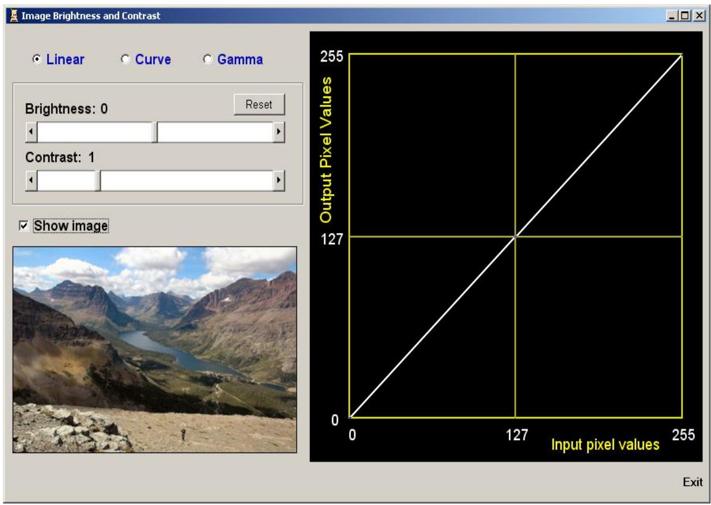

Figure II.3: Demonstrating the functional relationship between input and output pixel values for changing the brightness and contrast of an image

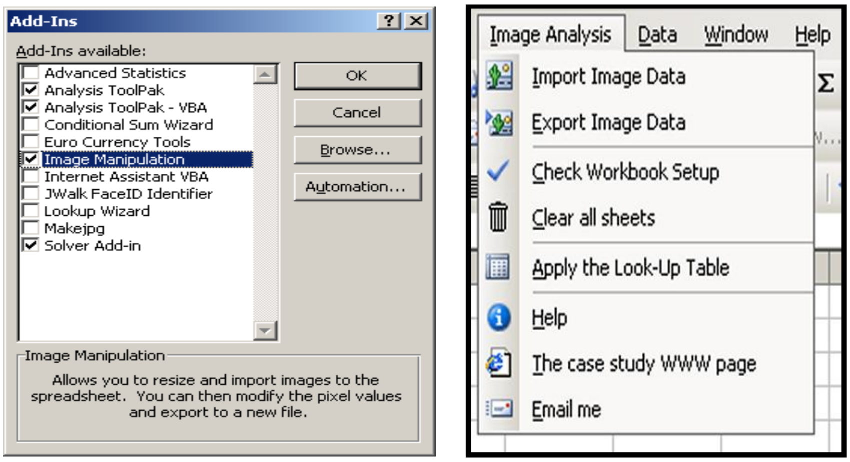

Figure II.4: The instalation of add-in and the new menu item

When installed, the add-in creates a new menu item as in figure II.4. There is a built-in help file, accessible from the ‘Help’ menu item.

The range of mathematics that can be explored in this way is broad:

- Brightness: **Straight lines**.
- Contrast: **Straight lines and curves**. e.g. Image Gamma : $255\left(\frac{x}{255}\right)^{\gamma}$
- Monochrome, colour casts, tints: **Arithmetic**
- Blur, Sharpen (unsharp mask): **Matrices**
- Translation, shear, stretch: **Linear/non-linear transformations**
- Distortion (e.g. ripple effect): **Geometry, trigonometry**
- Rotation: **Matrices**
- Noise: **Random numbers**
- Colour control: **Histograms, Statistics**
- Morphing: **Linear Interpolation**
- Edge-detection: **Finite differences**
- Filtering: **2-D FFT**

Example 2: Shapes in Images (Figure II.5)
In order to provide a way of modeling the mathematical shape of objects, a program has been written (DigitiseImage) that allows an image of an object to be imported and the edge of the desired shape to be traced. The user can click around the edge of this shape, each click representing one coordinate point. The image can if required be scaled so that the coordinates are in 'real' distance units. Once this has been done, the coordinates can be exported to Excel allowing a graph to be drawn, and suitable curves to be fitted, identifying the mathematical shape in the image.
This can stimulate a class discussion of where inaccuracies in this sort of modeling can arise, and hence what degree of reliability can be attached to the function identified.

Figure II.5: Digitise Image – Determining shapes in images

Example 3: Modeling a Live Game with Excel: (Figure II.6)

This is an example of a class activity which starts by getting the group to take part in a simple game, then uses Excel to model the game.

In this game, called Heads and Tails, the audience is asked to guess the result from a coin toss, whether it is head or tail. Persons with the correct guess can continue to the next round, and the game continues until just one person is left, who wins the game. The mathematical interest concerns the number of rounds it takes to find a winner starting with a particular number of players. What is the statistical distribution that results?

One way of finding out is by simulating the game, programming it into Excel.

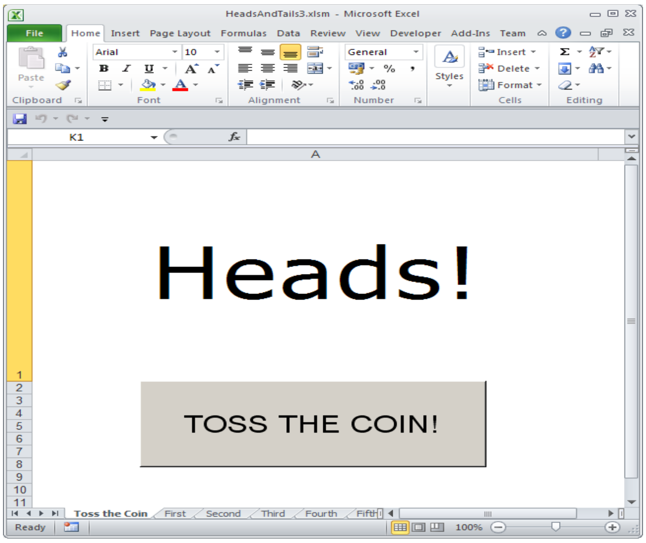

Figure II.6: Heads and tails game in Excel

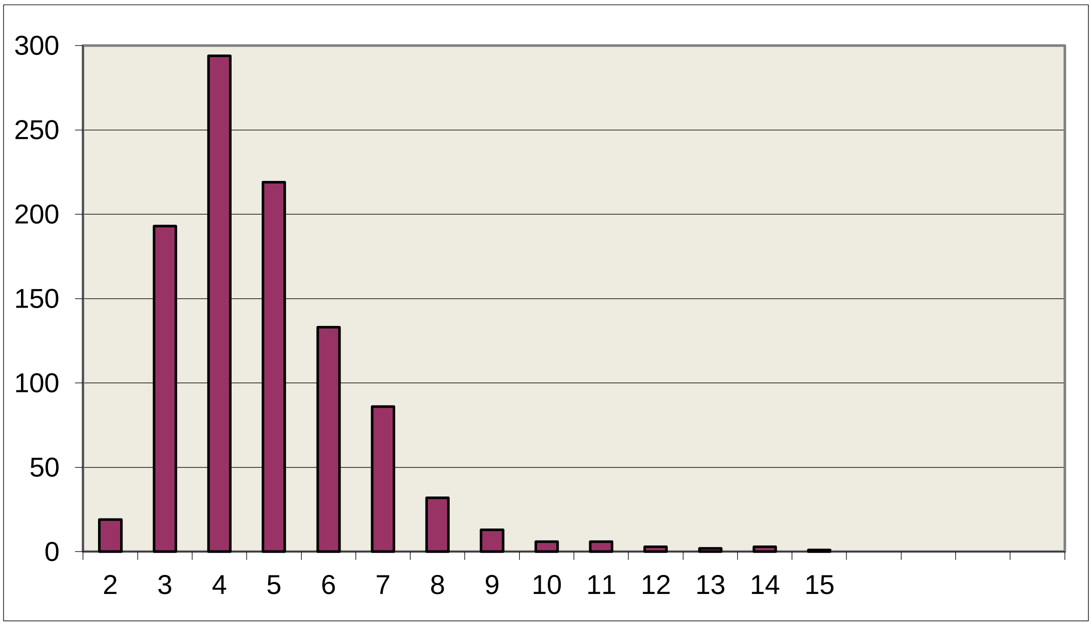

Figure II.7: The statistical distribution from 1000 games of 20 people

Using this program a distribution such as that shown in Figure 7 emerges, which simulates 20 people playing the game 1000 times. In nearly 30% of games a winner emerges after 4 rounds.

Example 4: Digital Signal Processing (Figure II.8)

A further illustration of the versatility of Excel as a modeling tool is in the implementation of a digital filter. It can be shown that the following difference equation will provide an effective band-stop filter for frequencies of 60Hz, given a pre-determined sampling frequency.

$$y[n] = 1.8523y[n-1] - 0.94833y[n-2] + x[n] - 1.9021x[n-1] + x[n-2]$$

The Excel worksheet shown in the screenshot below is of three sine waves combined – a high frequency component at 200 Hz, a low frequency component at 18 Hz and a central frequency. The slider allows the exact frequency of this mid-range component to be varies between 50 and 70 Hz. It is only within a few Hz of the frequency the filter is designed to eliminate that its effect can be seen – as shown in the screenshot. If the frequency is change by even 3Hz the filter’s effect is negligible, showing how well designed the band stop filter is.

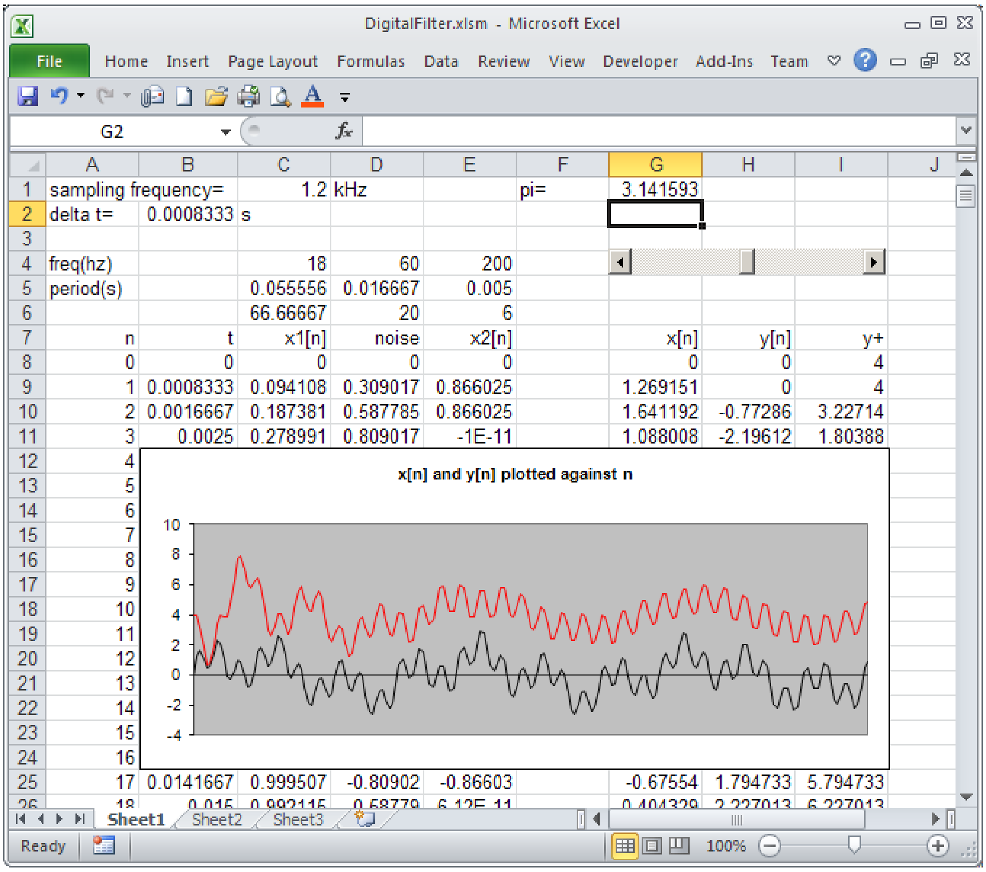

Figure II.8: Digital band-stop filter

III. Mathematical Modeling at MA ITB : Bringing real world problems into class room activities

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III.1 Introduction

Mathematical Modeling course has been successfully thought at the Department of Mathematics Institut Teknologi Bandung (or MA-ITB) for more than ten years. The essence of the course is to bring real world problems into class room activity so that the students will get a first hand experience in real world problem solving. In this paper we will discuss about Math competence and job market of graduates, facts and challenges of Math graduates, the Mathematical Modeling course at ITB, Industrial Mathematics activities and the conclusion.

III. 2 Shift of Paradigm in Teaching of Math

Motivating students in math is not an easy and short term task for math teachers. It is commonly understood that motivation comes from seeing relevances and experiencing real world problem solving. Currently there are many discussion in setting up the Industrial Mathematics society in universities in USA, Europe, and other countries. These could urge changes in paradigm in math teaching as follows:

- Conventional vs *Current & future challenge*
  There could be pressure from inside university or outside, such as economy's demand.
- Inward looking vs *Outreach looking*
  In inward looking, teacher is used to not looking relevance of subjects to real life problems
- Math for math vs *Math for world*
  Now math is always needed useful for solving real world problems.
- Teaching with "no rellevance" vs *Teaching with relevance that motivates students*
- Work in isolation vs *Work with other disciplines*
- Less respected by community vs *Work with community, respect and reward will come naturally*
- No access to industry vs *Building bridges to industry*
  Our responsibility as a mathematician to build a bridge to outside. We cannot expect people from other disciplines to come to us with clear-cut math questions
- Limited job markets for graduates vs *Wider penetration to job markets*

When students come to our university, ITB, and study for four years, they will get the degree and start to find jobs to build their career. Most of the time, they will get jobs which are not specific for mathematician, the jobs that can be fulfilled by graduates from any disciplines. In other cases when the fresh graduates got their first assignment in their job and are asked to solve the problem, they realize that none of their knoledge and skill from their study can be useful to answer the question, even to identify the problem. Unfortunately, there are still graduates remain jobless few years after their graduation.

Many of us assume that we know the required competences in job market from our point of view as mathematicians/lecturers. We forget that the competences are measured by stakeholders in the job fields. In Carnevale*et al.* (1990), the jobseekers are required to have competence in order to win the competition in the Job Markets, which are following:

- learning to learn;
- reading, writing and computation;
- oral communication and listening;
- problem solving and creative thinking;
- self-esteem, motivation and goal setting;
- interpersonal skills, teamwork and negotiation;
- organizational effectiveness; and
- leadership

We need to know how big the job market for math graduates. In fact, less than 10% of our graduates go to teaching jobs at universities or high schools. For the other part of graduates, most of them select job markets which are open to more general disciplines so the competition is considerably stiff. On the other hand, Math curriculum does not provide the students with soft skill to penetrate to new job market. The best math graduates may be suitable for jobs in Research and Development (R&D) division. However this is not yet available in most of industries in Indonesia.

There are common negative impression of society toward Math subject and community. Among others are scary, difficult to understand, boring, useless, and it does not give financial benefit. Those are simply not true. We need to show to the society that a lot of things can be done using Math because it can simplify complicated problems; it is easy if it is understood properly. Math is very useful for any problems, and it is the only discipline with wide range of penetration to other disciplines.

The following are facts and challenges being faced by mathematician in the country:

- Increase of complex problems in industries & societies
- Dependent on foreign and expensive technologies
  From our experience in doing research with oil companies in Indonesia, we can develop our own software which usually cost very expensive in buying the licenses.
- Unpreparedness in global competition
- Too late to wait for industries to build their R&D
- It is necessary to find a breakthrough in accelerating the role of math in industries
- There exist commonly low student's motivation due to unanswered questions: how, where and when the math subjects are used and useful
- The nature of mathematician to isolate themself from and to avoid taking roles in dealing with real world problems
- Many instructors assume that all math (they teach) are *applicable* with no effort of giving real examples
- No significant *critical mass* of mathematicians having real world problem solving experiences in most universities

- In a good point, there is significantly number of math alumni with good career in job markets. They come to us and bring problems from industry .
- Involvement of students in industrial activity may contribute to the expansion of job markets for math graduates

Why and when Math is needed? The answers are including below:

- J.L.Lion (1994): "*Mathematics helps to make things better, faster, safer, cheaper* by *thesimulation of complex phenomena, the reduction of the flood of data, visualization"*
- Math has deep penetration in all aspects of world problems as well as in all other knowledge
- Math is needed where ever dealing with complication

III.3 Mathematical modeling and simulation in industry and environment

The role of mathematical modeling is to understand, to predict, to optimize, to control the systems and to help to make things better, faster, safer, and cheaper. The summary of math modeling process can be seen in figure III.1.

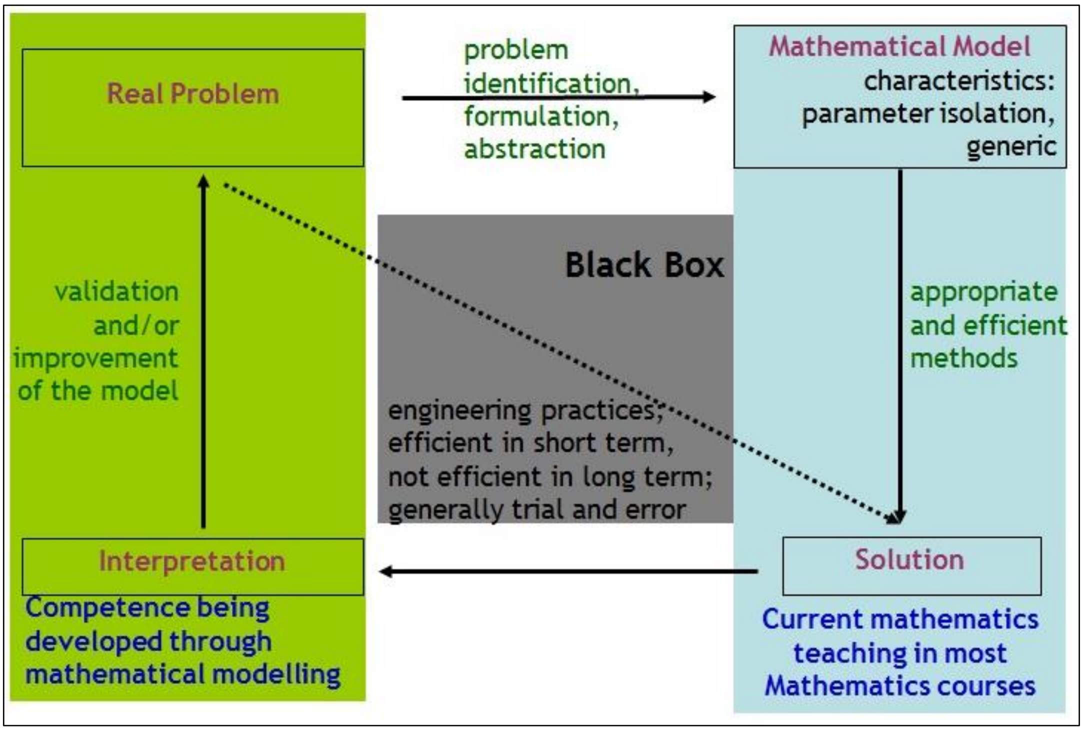

Figure III.1: Nature of Real World Problem Solving

Real world problems can be found from everywhere. The question is how we bring this problem to math's world. In reality the nature of the problems is not well posed, and even not provided with enough information. In the case that data is available, it could be useful to start with understanding and analizing the data. It can be assured that the skill of problem identification, formulation and abstraction to get mathematical formulation of the problem is nothing to do with how much math knowledge you have, even does not require math at all. If the problems are complicated, just simplify and simplify. The skill needed here is a competence that can be learned by everybody. Unfortunately, this soft skill is missing in the learning process in the classroom and the curriculum.

Having math formulation of the problem in the box on the right of figure 1, we use all tools, such as methods, concept, theorems and their proofs, in math that are taught in the classroom. However, if there is no attitude at the beginning in understanding the soft skill explained in the last paragraph above, many students can be frustrated due to their unanswered questions about the applicable and usefulness of the math tools being taught. Currently, we have seen in the curriculums from elementary school to higher education is the process only in the blue box on the right. We teach math without any interest in relating the process in other boxes.

Another issue is the relevance of math solution to the real world problem or the interpretation of the math solution. It may be done in every step of the processes, when we do the problem identification, the formulation or in solving the math problem. Having this relevance, we should compare the solution to the real situation happening in the real world, whether they are in lineor not. If not, we need to revise the process from the beginning and so on. From our experience, those processes can be done in all levels of math education.

In the other hand, the common practices in engineering is when the problem is solved directly by try and error. They do not go through mathematical modeling or abstraction, so as consequently the solution is not efficient for long time period.

We need to agree that math modeling is easy and fun. How do we get the problem? Problems are everywhere, what we need is just look around in our own places. It is not to be worried necessarily. As mentioned before, understanding real world problems doesnot need much math skills. This is more like an attitude to understand the problem. If the problem is too complicated, we just need to simplify and simplify again up to we get a model.

One that we should aware of is that students have somethingwhat we do not have as teachers and older persons. Students have motivation and good attitude on solving problem so we can explore them. Practically there is no prerequisite for students to start modeling. Certainly if it is high level of problem, they need some higher level knowledge. But for starting the process of modeling, there is no need to have some prerequisite.

III.4 Activities in math modeling at ITB

Activities in math modeling at ITB cover varieties of programs from math modeling course, student modeling activities, Industrial Math Week, join-multidisciplinary research with industries and society, and some collaboration for accelerating the role of mathematician in real world problems.

Math Modeling Course at Department of Mathematics (MA) ITB is run by eight of our staff: K.A. Sidarto, R. Hadianti, A.Y. Gunawan, N. Nuraini, N. Sumarti, J. Naiborhu, S.W. Indratno, E. Soewono. This course goes as follows.

- The idea of *Mathematical Modeling* is simply "bringing real world problems into class room activities". Problem solving process in real world is simulated as real as possible in the class room activities.

- In the first day of class, students are divided in groups, 4-5 students each. Each group is assigned different problem which are coming from industries, labs around ITB or from our environment.
- Each problem is assigned in short statement, not *well defined*, and in most cases only in the form of titles. This represents the situation in work fields in which the assignment is not provided with complete description. From our experience, sometimes the source person does not know already the question to be answered, but he/she only knows the real situation of the problem.
- During the first 4 weeks, students should understand the problem, find detail information, talk to resource persons, isolating and then formulating the problem in a workable form. The role of instructors is to assist the group in the discussion process. At the end of the 4th week, each group gives presentation in a seminar. At this session no math formulas are allowed to show. They only have to formulate the problem in a workable form.
- In the next 4 weeks they have to construct (simple) math model, find solutions and interpret the solutions. The results are presented in the second seminar.
- In the next 5 weeks they improve the model, validate (if possible), elaborate the results and relevance to the real problems, showing the advantage and disadvantage, and present it in the final modeling seminar.

In Appendix III.A we can find topics that have been discussed in the Math Modeling course years 2011 and 2012. These topics are from the people who come to us about their problems or from our observation in our surrounding environment. The topic can be very serious problems from industry such as "Water Pricing in Cascade Reservoir in West Java", "Mass flow rates in nuclear reactor channels, "Temperature distribution in deep sea pipeline", or "Material strength in geothermal separator". This can be problem from our surrounding place such as "Analysis of the management of public transport in Ganesha Street in Bandung", "Flood estimation in DayeuhKolot in Bandung", or "Model for Ganesha – Jatinangor Transportation cost for ITB staff". This can also problem which is general or popular issues, such as "The ability of cats to land safely","Are cats ever drink?", "Walk or run. Which one do you choose?", or "The phenomenon of Paul Octopus: is it myth or fact?"

In Appendix III.B, the result from questionnaire distributed to the current students of Math Modeling course is shown.In the question about the first reaction in the beginning of the class, some of students get more interested in the lecture (34%), but some of them were confused (12%). The most unpleasant experience for some students is finding data and creating models (26%) or doing some programming (9.7%). The most pleasant experience for some students is when they can solve successfully all cases of the problems (15%) or looking for data and developing a model (13%) which has been claimed also as the most unpleasant experience for some students. The knowledge that can be obtained for some students is something from outside the classroom (25%). The skills that can be obtained for some students are how to communicate in the presentation (42%) and how to work with computer programming and application (12%). Values that can be obtained for some students are how to appreciate others (29%) and being more patient and understanding with others (16%). After attending this course, most of the students become more aware of the applicability of math in solving problem (72%) and being more confident (86%).

Having experienced with math modeling course, some selected students in groups are involved in the Mathematics Contest on Modeling (MCM) and Interdisciplinary Contest on Modeling (ICM) which are organized by COMAP (Consortium for Mathematics and Its Applications), is a non-profit organization whose mission is to improve mathematics education for students of all ages. Since 1980, COMAP has worked with teachers, students, and business people to create learning environments where mathematics is used to investigate and model real issues in our world.

Every year since year 2000, COMAP holds Mathematical Contest in Modeling (MCM) and Interdisciplinary Contest in Modeling (ICM). The participants are higher school and university students who work in teams to find solution of provided real-world problems uses mathematical tools. In ICM, the provided interdisciplinary problems involved concepts from environmental science, biology, chemistry, and/or resource management, operations research, information science, environmental science, and interdisciplinary issues in security and safety.In this contest, the students should understand, identify, formulate and solve the problems in only four days. Only with four day works, many students can come up with good results for relatively difficult problems and even won variety categories.

The participants come from USA, Canada, Mexico, Australia, New Zealand, United Kingdom, Ireland, Germany, Finland, Lithuania, South Africa, Jamaica, United Arab Emirates, Hong Kong China, P.R. China, Republic of Korea, Singapore, Pakistan and Indonesia. We can see that not many economies in APEC region compete in these contests. The results classified participants into Unsuccessful, Successful Participant, Honorable Mention, Meritorious, Finalist, and Outstanding Winner. The highest awards (Outstanding Winners) are dominated by participants from USA (more than 75% every year), and other winners come from PR China , Ireland, South Africa, United Kingdom, Canada and Singapore. Indonesia, represented by ITB since 2002, has highest achievement as meritorious winner which is shared with 10% - 18% of total number of participants including some top-league universities in the world.

Annually, we organize Industrial Mathematics Week where persons from other fields can give real world problems. In four days, students and staff are working together understand, identify, formulate and solve the problems. At the end, there are presentations of the results in front of the resource persons. See Appendix III.C.

Having these modeling activities, we need to have a place as the gateway to industries. In 1994, we built Center for Mathematical Modeling and Simulation (P2MS-ITB), a multi-disciplinary research center. There are two active research consortiums in this center, which are

1. Optimization in Pipeline Network (OPPINET), founded in 2001, sponsored fully by oil and gas companies, such as Chevron Pacific Indonesia, TOTAL E&P Indonesie, PT. Perusahaan Gas Negara (Persero), Tbk., POMA Companies (Badak NGL, Chevron Indonesia Company, PT. Pertamina Gas, TOTAL E&P Indonesie , VICO Indonesia), BOB PT. BSP – Pertamina Hulu and Badan Pengatur Hilir Minyak dan Gas Bumi.

Establishment of OPPINET motivated at the time of by a development plan of an integrated Indonesia gas pipeline network (Indonesian Grid) and the ASEAN Grid, and even the Asia Grid. Constructions and operation of pipeline are very expensive so they need to be optimized in such a way that the development provides economic benefits for shareholders and provide maximum benefit for society and the state. The optimization research performed by OPPINET will contribute to the reduction of economy's spending in energy and increase the revenue.

2. Financial Modeling, Optimization and Simulation (FinanMOS) was founded in 2009. The research is on financial mathematics and optimization problems. Our clients are including Garuda Indonesia airline and PT Telkom Indonesia.

III.5 Lesson learned

There are lessons to be learned from all of these:

- Interdisciplinary collaboration is effective in bridging the gap between math and real world
- Mutual benefit creates benefit and reward to math
- Students see how math contribute to industrial problems– opening new job markets to math graduates
- Do not expect text book math will directly apply
- Abstraction of industrial problems arising as a spin-off

APPENDIX III.A: Recent Math Modeling Projects

Topics from Math Modeling course in 2011 are following:

- Analysis of the management of public transport in Ganesha Street.
- Long term effect in genetic replacement of mosquitoes for controlling the spread of malaria.
- Microbial Enhance Oil Recovery (MEOR) in a sand pack.
- Mass flow rates in nuclear reactor channels.
- Temperature distribution in deep sea pipeline.
- Material strength in geothermal separator.
- Optimal pin configuration in nuclear reactor.
- Banking security system.
- Computer Graphic modeling.
- The phenomenon of PaulOctopus: is it myth or fact?
- Forensic Data for predicting the results of World Cup 2010.
- The ability of cats to land safely.
- Are cats ever drink?.
- Stereovision problem.
- Weather problem.
- Treatment problem of Dengue patients.
- Brazil Nut Effect.
- Optimization in Fractional Ownership Aircraft.
- MDGs Indonesia: which should be focused?
- Prediction of the effect of global change in the increase of disease outbreaks.
- Optimal distribution of sea cargo to avoid the gasoline shortage.

Topics from Math Modeling coursein 2012 are following:

- Multiphase problem in gas distribution network
- Sensitivity analysis of PT.Garuda Indonesia flight scheduling
- Baggage Handling System in Airport
- Walk or run. Which one do you choose?
- Brazilian Nut Effect – The Series
- Car parking control
- Flood estimation in DayeuhKolot
- Revenue estimation of SPBU COCO
- New domestic routes of Garuda Indonesia
- Stability of Air Field Capacity
- Model for Ganesha – Jatinangor Transportation cost for ITB staff
- Predicting the traffic condition at Pasupati bridge for the next 5-10 years
- Stock Keeping Period in retail business
- Comparing three methods for milk homogenization
- Prediction of electronic waste in Bandung
- Strategy for production improvement for Indonesian Bamboo Society
- The Leaves of A Tree
- The Melting Arctic
- Train Problem
- Water Pricing in Cascade Reservoir in West Java

APPENDIX III.B: Result from questionnaire

QUESTIONER (Math Modeling Class 2012)

1	How did you react first to the given task at the beginning of this lecture	%
	More interested in this lecture	34
	Happy	8.1
	cannot imagine what model will be made	6.8
	To be challenged	6.8
	Enthusiastic	6.8
	Curious	4.1
	Afraid	4.1
	It looks cool	4.1
	So so	4.1
	feel burdened	2.7
	Good to getting involved in the application of Math	2.7
	Confused	12
	Expecting very complex and difficult task	1.4
	It seems simple	1.4
	Not expecting the given topic	1.4
	Total : 74	

2	Tell us your most memorable experience (the most pleasant and unpleasant) in this course.	%
a.	Unpleasant experience	
	Find data and create models	26
	Busy scheduled supervisor and teammates	11
	Programming	9.7
	Integrating all thought from teammate	6.5
	Contracting model valid to the reality	6.5
	Panic when giving the presentation	6.5
	Reading the literature	4.8
	Difficult to understand and solve the problem	4.8
	To derive a formula from the case	3.2
	Get stuck in formulating the model	3.2
	Adjusting the formulae of some physic laws with the model	1.6
	Looking for things that have not been studied	1.6
	Non-cooperative friend in a survey	1.6
	Simple thing in real life needs Math	1.6
	Understanding English written source	1.6
	Unhelpful supervisor	1.6
	Not enough information in concluding	1.6
	Invited to OPPINET	1.6
	Writing the report	1.6
	encourage each other when feeing down	1.6

	Given undesired topic	1.6
	Total: 62	
b.	Pleasant experience	
	Solving all cases successfully	15
	Looking for data and developing a model	13
	Working in a group	13
	Listening other group's presentation	13
	Working with lecturers from other department	11
	Find the formulation of the problem	9.1
	Deepen knowledge on modeling	5.5
	When the presentation is end successfully	5.5
	Got a compliment from the supervisor on the progress work	3.6
	Integrated all thought	1.8
	Got new experience	1.8
	Got a supervisor which is kind and cool	1.8
	Energy conservation law	1.8
	Conducting simulation and practicing the presentation	1.8
	Be able to work with Workplace and Matlab	1.8
	Doing all processes in modeling	1.8
	Total: 55	
3	What did you get from this class along the way?	
a.	Knowledge	
	Learn from out of the classroom	25
	Knowledge on our topic and other topics	15
	Turning a real world problem to math formulation	12
	Math application on daily life	12
	Methods on optimization	10
	Markov chain	5.1
	Theories, process and mechanism of modeling	5.1
	Mechanics on walking and running	3.4
	Realization that Math is useful and applicable	3.4
	Public knowledge on water price	1.7
	Small things from our surrounding place	1.7
	How to find the real size from the picture	1.7
	Market research and business plan	1.7
	Knowing that math can work with other field	1.7
	Total: 59	
b.	Skills	
	How to communicate in presentation	42
	Collaborate with others	24
	How to work with Matlab/Maple/programming/MS Power Point/MS Excel	12
	How to simplify problem	6.7
	Thinking systematically	4.4
	Data mining and stress management	3.3
	Mechanism in solving technic	2.2

	Enhance creativity and power of thinking	2.2
	Independency	1.1
	Writing a report	1.1
	Total: 90	
c.	Values	
	Appreciating others	29
	Responsibility and discipline	19
	Collaborate with others	19
	Being patient and understanding with others	16
	Comprehensive on solving problem	7.2
	Trust on each other	3.6
	Interaction with the lectures	2.4
	Attitude and language in presentation	1.2
	open to other's thought	1.2
	Take care on environment	1.2
	Total: 83	

4	Is there any change of attitudes after the class?	%
	Aware of applicability of math in solving problems	72
	To be encouraged and wanting to learn more math	7.8
	Solving problem systematically not only with observation	6.3
	To solve complicated problem can begin with simple things	4.7
	Be able to analyze a problem	3.1
	Math is difficult if it is not well understood	1.6
	Unselfishness and be more patient	1.6
	More sensible on others	1.6
	Modeling problem from other field from math	1.6
	Total: 64	

5	Are you more confident after attending this course?	%
	Yes	86
	No	14
	Total: 63	

6	What is your suggestion on this lecture?	%
	More time in presentation	23
	Presentation is conducted in front of all students, instead of separate classes	15
	More interesting topics	13
	Topics to be assigned are in the interest of the students	8.3
	Keeping the schedule of supervision	6.7
	The results is published in poster	3.3
	Mark individually rather than in group	3.3
	Open for problems from the students	3.3

	Schedule is announced earlier	3.3
	More efficient in presentation time	3.3
	More individual task on summarized all presentation	1.7
	Make the topic as the final assignment	1.7
	More applicative topics	1.7
	Further work on the model in order to have useful model	1.7
	Format of the report is known earlier	1.7
	Example on lecturer's presentation	1.7
	Publish the results in international conference	1.7
	Need advices on presentation	1.7
	Keeping the rules in presentation	1.7
	Supervisor needs to optimize the student's work	1.7
	Total: 60	

APPENDIX III.C: Math Modeling Projects from IMW

IMW 2010

- Flow assurance for oil/gas production in deep-sea pipelines
- The Long-Term Effects of the Lymphatic Filariasis Medical Treatment
- Analysis of aircraft fleet requirement in a fractional aircraft ownership program
- Development of zero-water discharge technology and nitrifying bacteria application in the nursery phase of giant freshwater prawn *Macrobrachiumrosenbergii* de Man
- Optimization of water supply allocation in cascade reservoir in citarum river (saguling, cirata, jatiluhur)


IMW 2012

- Fishing Rod Design
- Oil detection using electromagnetic prospecting tech
- Piped Water Cooling of Concrete Dams
- Pollution modeling
- Corrosion inhibitor
- Brazil Nut Effect
- Environmental Engineering topic

IV. Teaching Mathematical Modeling at the High School Level

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Meridian Academy, Brookline, MA, USA
School with descriptions of interdisciplinary courses: www.meridiancademy.org
Modeling Materials: www.meaningfulmath.org
Open-ended pure math investigations: www2.edc.org/makingmath/mathproj.asp

IV. 1 Introduction

Hello and thank you so much for inviting me here today. This conference has provided a wonderful opportunity for me to work with my colleagues here at ITB and I am grateful for that work and the chance to speak with all of you and to extend the conversation about the purposes of education and mathematics education in particular. I hope that this is, in fact, just the beginning of a long conversation since the work we face is great and ever changing. As will become clear from my descriptions, my teaching style is not generally lecture based, so this is an uncharacteristic mode for me. If you have questions about my comments, do please raise them and I would be happy to diverge from my script to clarify or elaborate on anything that you wish to hear more about.

IV.2 How I became interested in these teaching and learning goals

My father is an accountant who loves mathematics and when I was young, he would do math puzzles with me. I recall this addition problem, called a cryptarithm, that he gave me when I was little. See Fig.IV.1.

Fig IV.1: A cryptarithm

The challenge was to figure out which digit each letter had to represent. You could not use the same digit for two different letters. What made this a challenge was that it was not an exercise like those I did in school. It did not come with instructions on how to approach the problem or help me know which of the skills or ideas that I had already learned were going to be useful. I was working without a guide or directions. My father's ability to instill in me the confidence and eagerness for working in such a way has influenced me as an educator as I help students tackle challenges that are more open-ended, and risky, in the sense that there is not one predetermined path to success.

I only had two math teachers in high school, both fabulous. One loved pure mathematics and would get teary-eyed talking about the great mathematicians, like his favorite Carl Friedrich Gauss. The other loved applied mathematics and had us write research papers on how math was used in the real world. One year, I wrote about how electronic music is produced (then a difficult feat that required a room-sized computer just to make single notes at a time).

I went to Yale University and studied biology and computer science. Although I was a science major, no one ever required or suggested that I take a statistics class. The goal was not to help me do science actively, it was to stuff into my head all of the interesting science facts and concepts that my professors had discovered. That, however, did not help me develop the habits of mind that actually produce effective new science. For that, you need to not just answer questions that are already known, you need to ask new questions about remaining mysteries, you need to be able to design new experiments informed by statistical skills that will yield meaningful and significant results. I had one professor, who clearly considered himself a renegade, a trouble-maker, who taught us to look at published research papers and to question their claims carefully. This was eye-opening to me that peer-reviewed articles might not be all that good.

When I graduated from college and started teaching full time, I was teaching a traditional Algebra I math class from a traditional math text to 9th graders. It had problems like in Fig.IV.2.

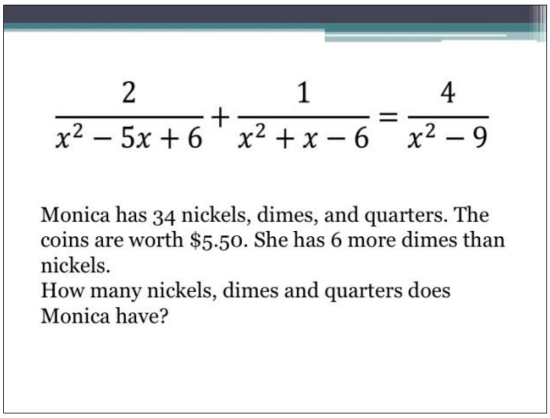


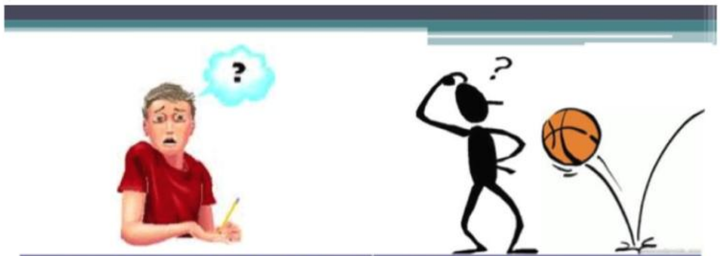
Fig. IV.2 : Rational and coin problems

I had enjoyed such work as a student because I viewed every activity as a puzzle needing solution. I liked the challenge and did not care if it had any purpose. I quickly discovered that most of my students did not see these tasks as intrinsically pleasant, as fun for their own sake. They asked “When am I ever going to use this?” I replied, quite seriously, “Why, in Algebra 2!” In other words, my answer to when these fake skills might be needed was in a different course that we had created that also required useless skills.

At the time, I did not yet know how to teach math so that their question had a real answer. I started to reflect on my own college studies and on what mathematics I needed in my biology studies and my computer science classes. Clearly algebra was important. Clearly statistics was important. Clearly calculus and geometry were important, but the classes that we taught did not help students see these connections between math and other subjects or know how to apply mathematics in a meaningful way.

The analogy that I now like to make is the following. Imagine that you want to teach a child basketball. You train them to dribble the ball with one hand. You have them practice shooting a standing set shot. You make them pass the ball to a target on a wall. But, you never tell them that there will be an opponent who is going to try to swat the ball away from them. You never mention that they need to pass to a moving target. And, you never explain that there is strategy that needs to be considered and re-evaluated as a play develops. That they need to constantly re-think what the right action to take might be: pass, move without the ball, drive to the basket, shoot the ball. Put them in the game and they will stand there bewildered, useless.

That is how most math classes in the United States are taught. At some point, we created something I call "school math" that is a lot like that basketball training that I described. See Fig. IV.3.



Confused Student in "School Math"	Confused Basketball Player
Ask an interesting question?	Choose a play?
Introduce a variable?	Decide on a defense?
Interpret a graph?	Rebound?
Solve an equation?	Pass?
Simplify a problem?	Shoot?
Understand what you don't know?	Determine a strategy?

Fig.IV.3: School Math and Basketball training
(http://www.washington.lib.ia.us/images/decorative/question-guy,
http://www.topendsports.com/image/clipart/basketball/bouncing-basketball.gif.php)

We taught many skills and invented exercises that helped students practice those skills. But we never told them that there was a game, that mathematicians really did two things with these skills:

1. They discovered and invented new patterns, relationships, and properties. That is, they did pure theoretical research that involved surprises, paradoxes, beautiful symmetries, and unexpected connections, that was limitless in its range.
2. They also used all of these discoveries and tools to ask and answer questions about their world, to make science, economics, psychology, urban planning, and so many other areas of learning more effective. What students were not learning was that mathematics, as one discipline, was a window on their world. See Fig. IV.4.

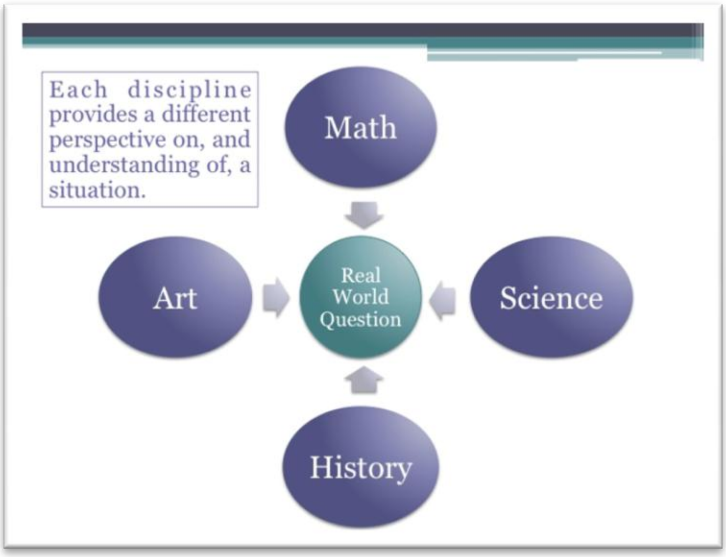


Fig. IV.4: Math to real world question

In response to my students’ frustrations, I gradually developed goals for my classes that would give them the opportunity to do mathematics as it is really carried out, to be junior mathematicians. I then worked backward from those goals and figured out what skills, understandings, and habits they needed in order to reach those goals. Lastly, I developed lessons and longer experiences that would help them master those needed foundations. I have created curricula to help teachers think about how to teach both pure and applied mathematics and today I will talk about my work teaching students how to do mathematical modeling.

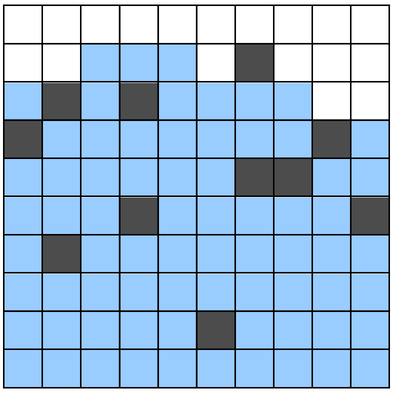
IV.3 What is Modeling?

Mathematical Modeling is the process of using mathematical tools and methods to ask and answer questions about real world situations. When working on a problem, you develop a mathematical model which is a representation of the situation of interest. Just as a physical model (such as a toy boat, a car being testing for aerodynamics, or a picture of an atom) represents an object, capturing some of its essence (appearance, proportions, its behavior), but also simplifying or neglecting other details (e.g., size, same materials, function), so, too, a mathematical model represents a situation symbolically, graphically, and/or numerically maintaining the essential aspects most important to your question and eliminating less important details. A model, as with a fashion model, is an ideal with the flaws or complications removed. This representation is then mathematically manipulated, using all of the techniques that we do teach in school, to discover something new.

IV.4 Why do we use models?

a. Understand.

Just as physical models have different purposes, so, too, do mathematical ones. One goal for a modeling question may simply be to **understand** a situation better for our own curiosity. The most open-ended and complex project that my 11th grade students do is work in teams for 5 weeks developing a model to answer a question that they think up themselves. I will share some of these questions during my talk to give an



A two-dimensional snow mound.

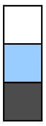


Fig. IV.6: How do soot particles affect the melting of snow mounds?

example of the breadth and sophistication of work that students can do and to highlight the different reasons for doing modeling. The question, *How do particles of dirt affect the melting of snow mounds?* was posed by students toward the end of a very snowy winter that we had in Massachusetts. See Fig. 6. It was May and the weather was warming, but there were still large heaps of snow everywhere covered with layers of black dirt. This group of students did not plan to do anything about the snow, they were just interested in whether the dirt did more to insulate and protect the snow or to absorb sunlight and melt it. They knew that the process of identifying variables, creating representations, working with those abstractions to generate new information, and determining the significance of that information to the original question would help them toward that end.

b. Optimize.
Another reason for modeling is to find the best solution to a problem. Mathematicians have developed many methods for finding the maximum or minimum value of a variable. Calculus and linear programming are examples of such powerful tools. For examples:
- How can delegates to the UN be seated to minimize tension (placing friends together and enemies apart)?
- How should the penalties for speeding fines be structured to generate the greatest revenues?

c. Design.
We also model to help us design physical objects and means for accomplishing a task. Since we typically do not want to design something poorly, optimization is usually a main goal here as well. For examples:
What arrangement of ceiling lights provides the most even illumination of a room?
What design of a movie theatre provides optimal viewing lines?

What stirring method most evenly distributes chocolate chips within cookie batter?

d. Predict.

Lastly, we model to make predictions. The lobster group worried that harvesting regulations might be set to leave only the minimum number of lobsters needed to replenish the stock. They developed a model for predicting lobster population changes over time. Using the model, they discovered that a too aggressive policy would result in a complete collapse of the population if any decreased birthrate or other perturbation reduced the stocks below the minimum level. The second question here is probably the most important modeling challenge facing the world today.

Why don't we just study the thing itself? Models can be ***cheaper, faster, easier, safer, possible.*** Some situations are not accessible – you can't actually visit the sun, but you can study its dynamics with a model – and some decisions are not easily repeated or revoked and so we want to test out possibilities ahead of time. Before we reintroduce a predator back into an ecosystem, we want a sense of the consequences. Before we dump more CO2 into the atmosphere, we want to understand the effect that it is going to have.

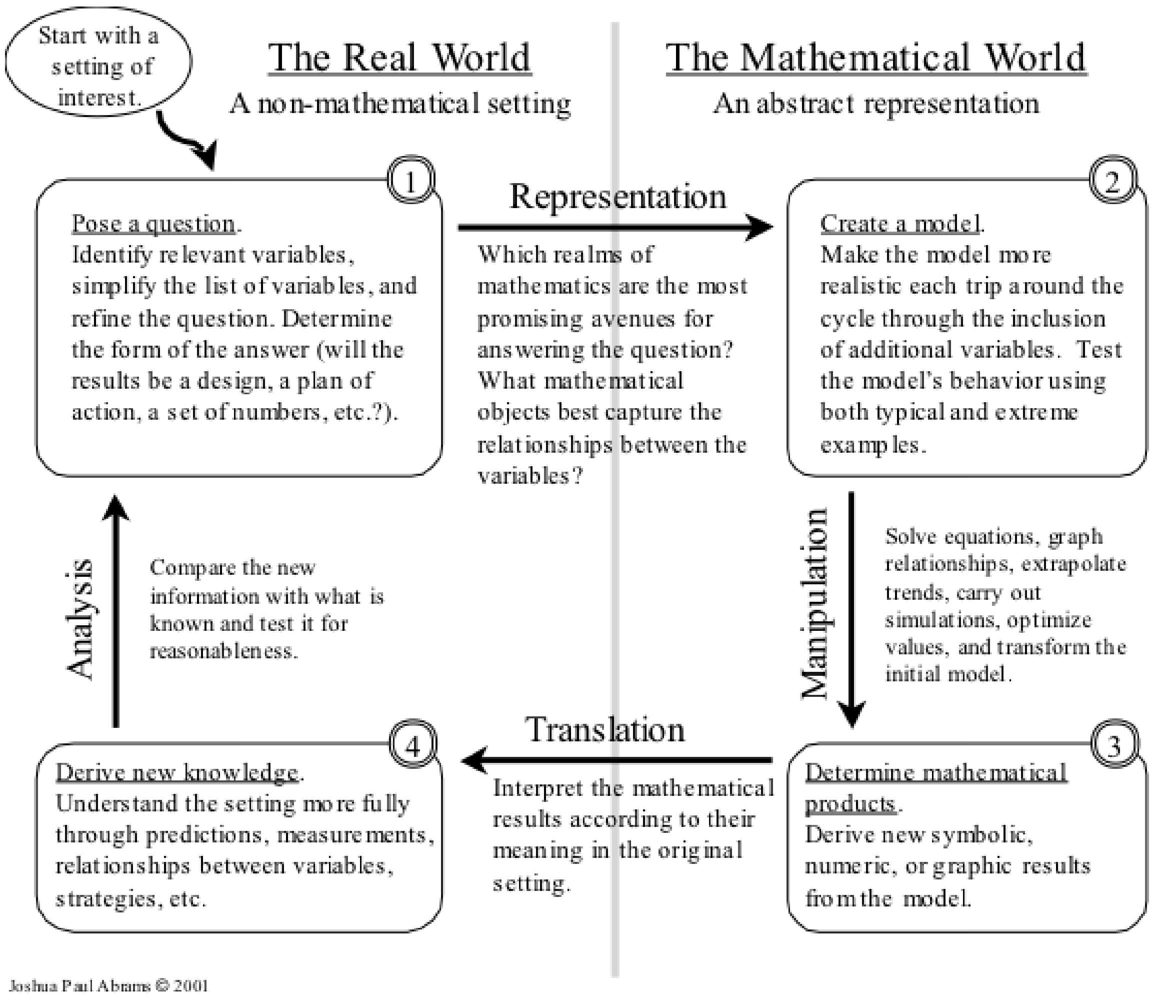


Fig. IV.7: The cycle of modeling process

IV.5 The modeling process.

The process of creating a model and using it is an involved one. Fig. IV.7 is a simplified diagram of a process for creating a model.

We start with a setting and identify a question of interest and identify relevant variables. We pick a small number of these and identify relationships between them, which we express mathematically. Once the initial setting of interest has been represented, it is then mathematically manipulated to discover new relationships inherent in the situation (this is where we use all of the math skills typically taught in school). We then identify a new plan of action by translating these new findings back into the context of the real problem.

Each stage involves many skills that need to be taught explicitly. My school teaches math and science in coordinated interdisciplinary classes. A goal of our curriculum is that students be able to create original models. What are the differences between these experiences and working on routine exercises and in what new ways do we have to help our students think and work?

Modeling work is (see Figure IV.8)

a. Unfamiliar and original. Each problem is different than previous ones. We need to help them embrace this uncertainty. One key is making learning less risky. We give our students the time to try things out, get stuck, revise their thinking and try again. Math is not actually a speed sport.
b. Memorable. Since students often pose the questions on which they are going to work themselves, they are highly motivated, care about the challenge, and retain the learning longer than if they were just memorizing for a test.

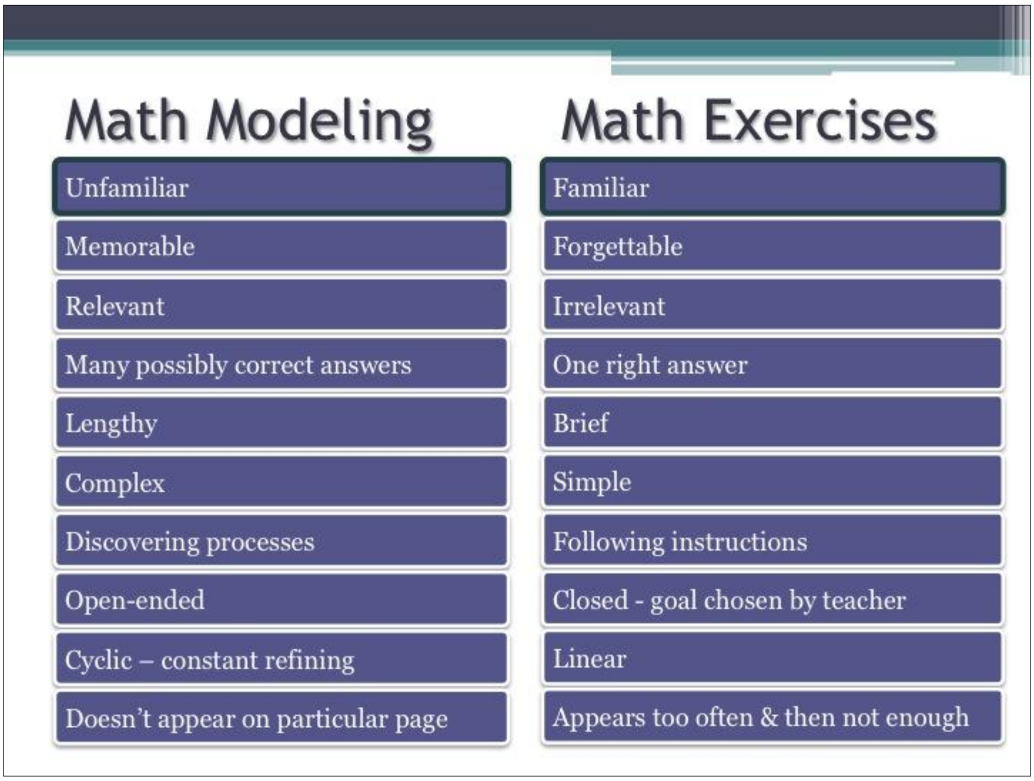


Fig. IV.8: Math Modeling vs Math Exercises

c. Relevant. The students at my school do not ask why they are learning because they continually see the usefulness of their learning. Students pose more difficult problems for themselves than we would ever do or which coming from us might seem boring. Because each student or group picks their own question, everyone gets to work on something that is interesting to them. Motivation remains quite high.
d. Not predetermined. There are many possible ways to address a question and the assumptions that we make can take our work and solutions in different directions. That means that the highlight of mathematics is not that there is one right answer but multiple perspectives that need to be assessed in the context that inspired them. Math can involve uncertainty. This makes assessing student modeling efforts a complicated task. Because there are so many things one can do correctly, we work on noting what students do pursue rather than specific approaches that we have thought of but which they may have not. They need to include steps, but good justifications of their choices and testing of the effect of those choices is more important than coming up with the same approach that their question may have led the teacher to think of.
e. Lengthy. Students need to learn how to work on a problem for weeks, how to break it down into sub-problems, how to get stuck and consider alternatives to getting unstuck. Many habits have to change for students to move from 15 minute problems to 15 day problems. We devote class and homework time to these efforts when they are underway. We check in with students to make sure that they are making progress and they maintain a log or diary of all that they do. We also teach them how to work in teams and make decisions collaboratively.
f. Complex and multi-faceted. Again, so much of this kind of work requires an ability to organize a lot of information and to connect different ideas.
g. Discovering new processes rather than following cookbook instructions. For example, the students who asked the lobster question needed to explore rates of change in the population and developed numerical methods related to calculus ideas that they had not previously studied.
h. Modeling problems do not have an end in the same way that an exercise does. One has to decide when you have enough information to move forward with a decision. Students learn that interesting problems are open-ended and that you can keep finding new directions to pursue. That quality is exciting. Being a true learner means understanding that there is always more to learn.
i. Real modeling problems do not come with instructions. So much of the work kids do, they know what is expected. Real life requires **diagnosis**, our ability to look at a situation and determine out of all of our learning which parts might be relevant, to try them out, and if unsuccessful, try a new tactic. Math exercises and tests often tell us which skills to use. The students who wanted to seat United Nations delegates near their allies (the real UN seats countries alphabetically) created a matrix showing friend and enemy relationships. They had not studied matrices in school, but they looked them up and learned the basics of matric operations well enough to realize… that they were not helping. They then went in a more geometric direction that was

fruitful. Modeling is about representation and these students considered different ways of turning their problem into a math one before they found a way that worked.

IV.6 What are the Goals of Education?

Why do we work so hard at it and put so many resources into it? Perhaps the main reason that people give is the development of skills needed for **employment**, to be able to contribute to the economy.

For me, just as importantly, I think education is about **citizenship**. If our students are living in a democracy, they have to be able to evaluate complicated issues and proposals and decide among them. This requires a much higher level of sophistication, especially mathematical and scientific sophistication, than it used to. If we are putting our fate in everyone's hands, we must give them the tools that they need to make wise choices. More than anything, I want my students to become informed, critical analyzers of information on general issues of public concern. To be able to reflect on the ideas and claims which they encounter and use math to understand them. And, **the only way to get good at using math to understand unfamiliar questions and new settings is to practice doing so repeatedly**.

I also think we learn because it is our intellectual history and because problem-posing and problem solving are fun and one goal of education should be increasing our capacity for happiness.

That the need for doing modeling is clear. Nonetheless, establishing modeling in schools involves a number of challenges. Once you talk about jobs, in my country at least, the arguments quickly turn to basic skills. Higher order thinking is not seen as basic. Just as no basketball team wins praise for turning out great dribblers, schools should not be asked to simply cultivate lower level skills. Dribbling in context, in a game, is what counts, and the same is true with math and all of the subjects that our students learn. There is no doubt that the kind of work I want students to be able to carry out requires very strong foundations in their math skills. The question is whether practicing these skills alone provides those foundations. The reality is that **the best way to really understand skills and to get good at them is to use them in new ways that require their repeated application**.

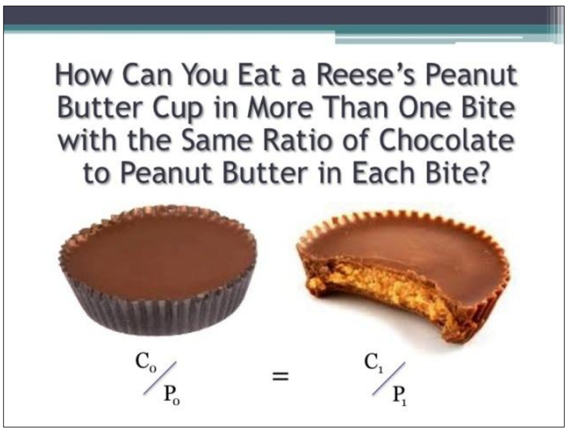


Fig IV.9: Rees Bites (http://skipthepie.org/nutrition-photos/,)

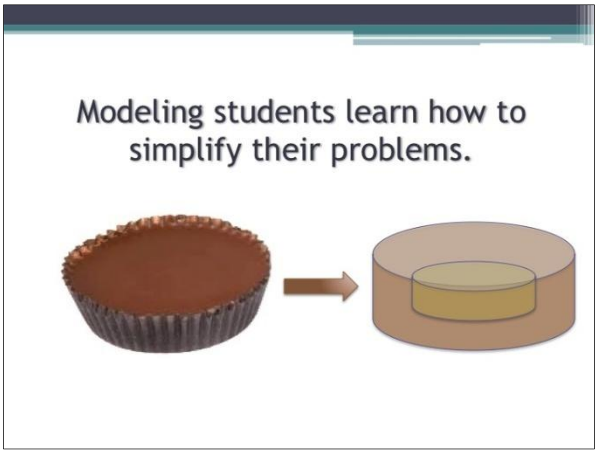


Fig. IV.10 : Simplify the problems

Let me give you one of my favourite examples. See Fig. IV.9 (http://www.health.com/health/gallery/0,,20306961_7,00.html). One of my student groups a while back asked this question. Now, this is not the most important question that humankind faces. The world will survive if it is not answered. But, my students loved it and wanted to solve it very much and quickly discovered just how hard it was. One of the skills that modelers need is to be able to simplify a problem and this group recognized that incorporating the ridges and angles of the sides were beyond their math background. They simplified the problem to a cylinder of peanut butter within a cylinder of chocolate being bitten by a mouth with a different radius than either cylinder. See Fig. IV.10.

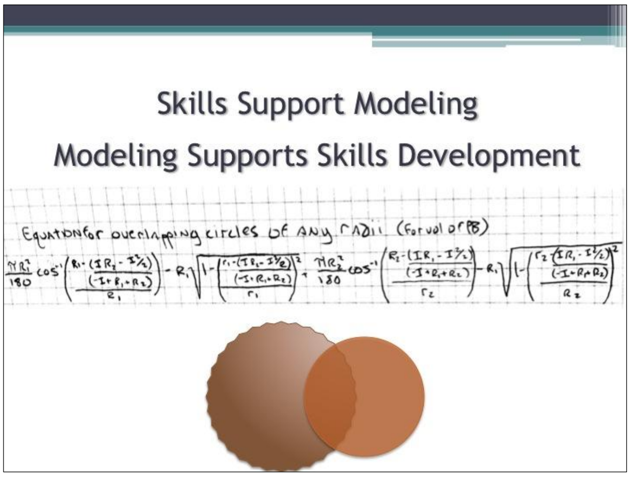


Fig. IV.11: A function for the ratio

They then developed a function to determine the ratio. See Fig. IV.11. This formula is not their final answer. It is one piece of the final function that gave the peanut butter to chocolate ratio for a bite of a given depth. Look at it. It involves rational functions. Variables and constants. Radicals and inverse trigonometric functions. They had to use conic sections and solve systems of equations. In short, pretty much every topic they had ever learned was wrapped into this one effort. Let’s look at that formula one more time. Now if I had proposed this problem, no one would have welcomed it. Something this hard had to come from the students themselves and when you give them the support and opportunity, kids will ask and answer questions much harder than a teacher could ever successfully inspire students to care about. They have ownership of their own learning and that is so motivating. See Table IV.1.

Adopting a Modeling Curriculum
Recognize that basic skills are best learned as part of more complex efforts.
Put more resources into professional development than curricular development.
Make interdisciplinary studies possible.
Match our assessments to our goals.
Make professional development similar to how we want our classrooms to function.
Avoid blame: celebrate past efforts and locate the need for change in our changing world.
Redefine what it means to be an expert.

Tab. IV.1: Adopting a Modeling Curriculum

There are a number of schools, although not a large number, who do this kind of work. Those who do get positive attention for their efforts, but, for reasons I will identify, there is not a lot of replication of these approaches. There are many wonderful curricular materials that have been published in the past 15 years with a strong modeling flavor and most have gone out of print fairly quickly. So, one hurdle in the United States is that while the government has supported curriculum development projects, it has not put the needed funds into professional development to help those curricula thrive. They are expected to survive in the market and since modeling is not required, sales end up low, and the materials do not thrive. They might spend $5 million dollars each on a dozen projects. I would rather they spend that on one and the rest on helping to implement it. We need to stop rediscovering the wheel and instead start attaching it to cars.

Nonetheless, one of the most talked about current trends in education in the US at the moment is STEM (Science, Technology, Engineering, and Mathematics). This is not a well-defined movement and some people interpret it as adding more engineering to the curriculum, and some think in terms of more interdisciplinary classes linking the four subjects as my school does, in particular at the middle school. Making genuine connections between the disciplines inspires and will require modeling and I hope that this new enthusiasm for STEM heads in the directions that we are all here talking about.

IV.7 Barriers in Mathematical Modeling teaching

One barrier to growth of modeling is that teachers teach what is assessed at the state or at USA level. Since states used timed tests to assess student learning, modeling activities that demonstrate a student’s ability to work overtime on complex problems is not going to have an easy time getting established as a core objective until we adopt better and more nuanced ways of assessing work and worry less about reliability and more about ambitious goals. If the only behaviors that we can reliably assess on a test are not good goals, then we have to let go of that approach.

Another barrier, as big as any, is teacher training. Teachers tend to teach the way they were taught. Since most teachers have not learned mathematics as an active discipline and have not engaged in modeling work themselves, they are not equipped to mentor students through this process. Modeling can’t be required until we have trained a large core of teachers equipped to teach it. I think most teachers would need many weeks of intensive workshops to reach the point where they could begin to do this work in their classrooms. Then they can teach as they have been taught.

There are psychological barriers for teachers as well:

a. US schools go through periodic waves of “reform.” The attitude of many teachers is one of resistance to change. In part, this is because they have been re-formed so often that they are cynical about the process that never gives them the time and support to prepare for the changes or a voice in the changes. In part, teachers may believe, without stating this themselves, that the idea that they have to change implies that all of their previous years of work were not any good. Not wanting to admit that, they reject the need for change. This is an attitude that has to be taken on directly by advocates of change. We need to celebrate past efforts, acknowledge the quality of the practitioners, and identify the reasons for change as coming from outside of that practice. We have to talk about the changing world, about education as an ever-evolving field, about the purposes of education that necessitate change. Our students are only going to be good learners if their teachers are learners as well. I once read an editorial in a math education journal in which the author claimed that it was unreasonable to ask a teacher to be able to change their methods by more than 10% a year and that a teacher should strive to change what they do by at least 10% each year. That 10% was their target and I think that any top down mandated changes should take that to heart in the planning process. Change takes time but should be evident.
b. Another supportive message that teachers will need to hear is that being the expert in the classroom does not mean having to know the answer to every question. If we are doing our jobs well, students will ask questions that no one knows the answer to yet. We have to be expert at showing our students how we would go about finding answers to good questions and be willing to get stuck, and explore options, in front of them. Teachers need permission to be confident facing a hard question. If they can show that that is where the fun begins, they will be able to lead their students into this kind of work.

IV.8 Examples

Engaging in the modeling process involves many skills, lots of writing, documenting your decisions with clear explanations, and it only works when one has time to think. My school has interwoven throughout our grade 6 through grade 12 curriculum a range of activities and assignments including problem sets, labs, papers, and projects that build up students’ ability to write about mathematics and to carry out ever longer modeling experiences.

These efforts fall on a spectrum from standard problems set in a real world context that only have one expected solution and where the students are prompted to use a particular topic, all the way to the projects I have described which start and conclude with a real world concern and for which the math is a tool but not the goal. I will share a few examples.

A Spectrum of Applied Mathematics
Level 1 – A standard math exercise with a real world setting.
Level 2 – An applied problem with multiple means of solution.
Level 3 – A setting is explored with multiple possible mathematics representations.
Level 4 – A model is provided and students are asked to modify and extend it. Given a tool, students need to find a setting that matches that structure and then apply it.
Level 5 – A setting is provided, but a model needs to be developed entirely by the student.
Level 6 – Students pose their own question and carry out the entire modeling process.

Tab. IV.2: A Spectrum of Applied Mathematics

The examples are following. We do a lot of activities to help students understand very small and large quantities so that kids can grasp the difference between a million or billion or trillion of some measurement. See Fig. IV.12 and IV.13. After practicing with estimation problems such as the number of grains of rice eaten in a year, students have to make an exponentially growing list of examples called an exponential ladder – we do these at the middle and high school.

Fig. IV.12 : Level 2

Exponential Ladders

Ground rules for counting rungs:
➢ Only those examples with correct bibliographic citations or calculations or labeled believably as common knowledge are counted in the below criteria.
➢ No more than 20% of the examples can come from one source (those in excess will not be counted).
Rung quality
➢ Clarity of example (exactly what is being measured is obvious)
➢ Accuracy of example (correct values on correct rungs of the ladder)
Overall Ladder Quality
➢ Ladder is ordered in terms of one clearly identified unit
➢ Extreme maximum and minimum examples are found
➢ Examples between those two bounds are filled in with few if any gaps
➢ Examples are drawn from a variety of domains (e.g., biology, everyday life, astronomy)
➢ Multiple examples provided for some rungs
➢ Originality
➢ Interesting comparisons
➢ Neatness and organization, ladder is typed.
Estimate Section
➢ Calculations for any estimated values are presented
➢ Conversion of researched data into the chosen unit is presented for at least one example.
➢ Estimates do not include unwarranted precision

Fig. IV.13: Exponential ladder

Younger students can do ladders of money, length, time, or mass. Older students do ladders with compound units like speed, density, pressure, or energy. This work reminds students that numbers typically need units and prepares them for dimensional analysis.

We teach students about many different functions: polynomial, rational, exponential, etc. See Fig. IV.14. To students at first these all seem like somewhat interchangeable shapes.

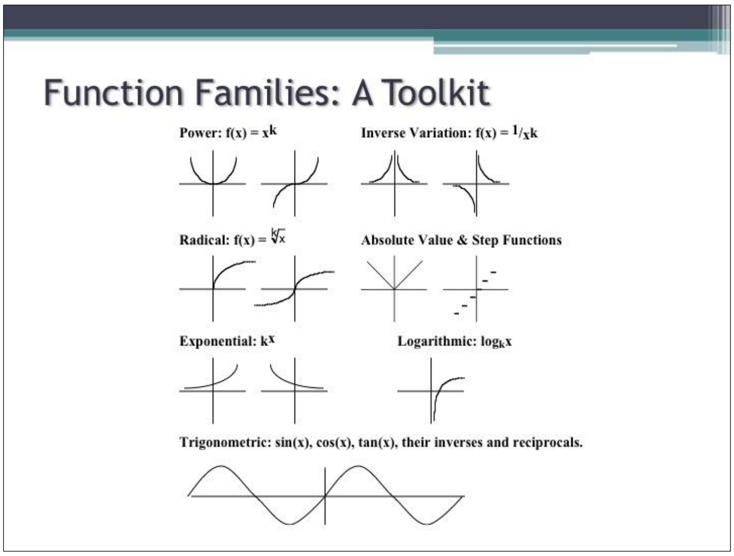


Fig. 14: Function families

Once we explore their different properties, they carry out informal "kitchen" experiments using everyday materials that are really physics experiments. See Fig. IV.15.

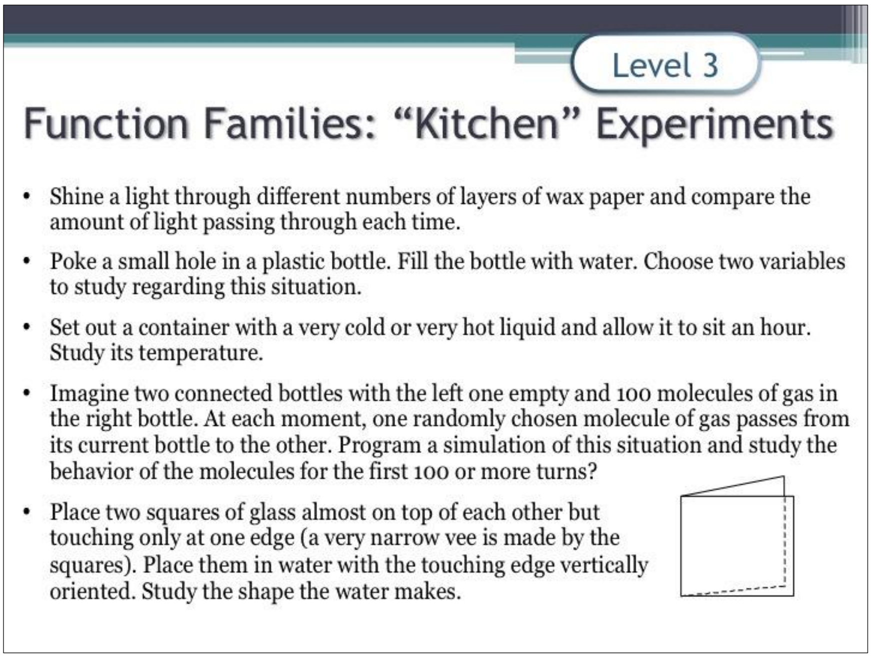


Fig. IV.15: Kitchen Experiments

Before they carry out their own experiments and analyses, we do one together as a class. See Fig. IV.16 (left). This one involves how much mass is needed to break a thin piece of spaghetti sticking out over a table edge. Once the students get data, they find that they can fit almost any function to the curve. We don't tell them which of these functions is right. They are all pretty successful if you want to interpolate new values. How do we choose? While they all describe the pattern of the data, when the students do the dimensional analysis only one ends up having units for the constants that add insight to the description. The inverse function has units that are related to torque and it is that bending force that is doing the breaking. See Fig. IV.16 right.

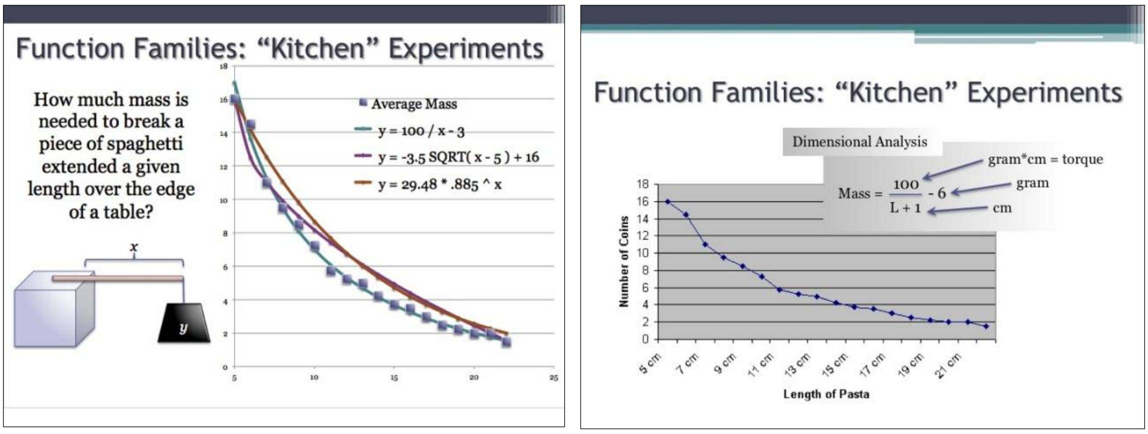


Fig. IV.16: Kitchen Experiments

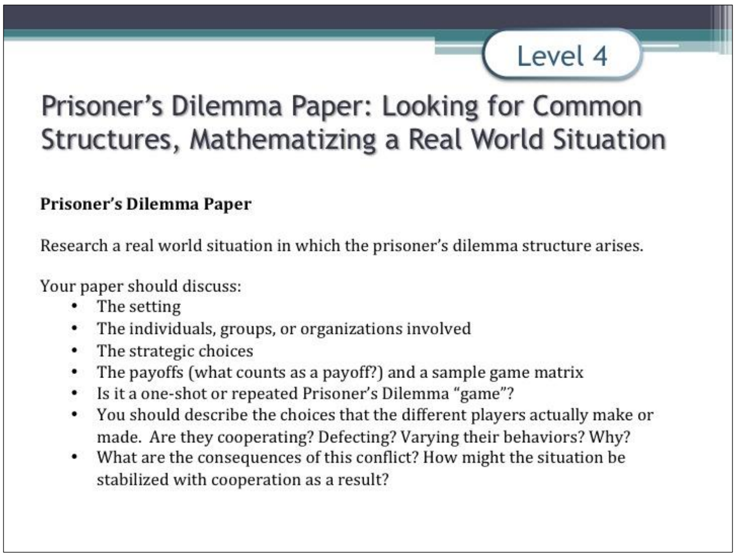


Fig. IV.17 : Prisoner’s dilemma paper

In one of our classes, students study game theory and learn about a famous, somewhat paradoxical, conflict called Prisoner’s Dilemma. They then need to find a real-world situation that exhibits this conflict and justify that they are really the same structure and use the model to better understand the conflict and to propose a resolution. Students have written about nuclear arms races, whether women should wear make-up, whether to get a vaccination, and much more. See Fig. IV.17.

There is an international event called the High School Mathematics Contest in Modeling (HiMCM) that has participation of over 400 teams from over a 100 schools internationally. See Fig. IV.18 (http://www.comap.com/highschool/contests/himcm/flyers/HiMCM2012.pdf). The teams come primarily from Asia and the United States. The teachers and students dong this work are a real resource for the modeling community. The contest provides a setting and a question, but the students have to work as a team to develop an original approach to the question. A sample of the question is in Fig. IV.19. Note that while it sets the stage for a problem, it does not say what to do or how to measure “security.”

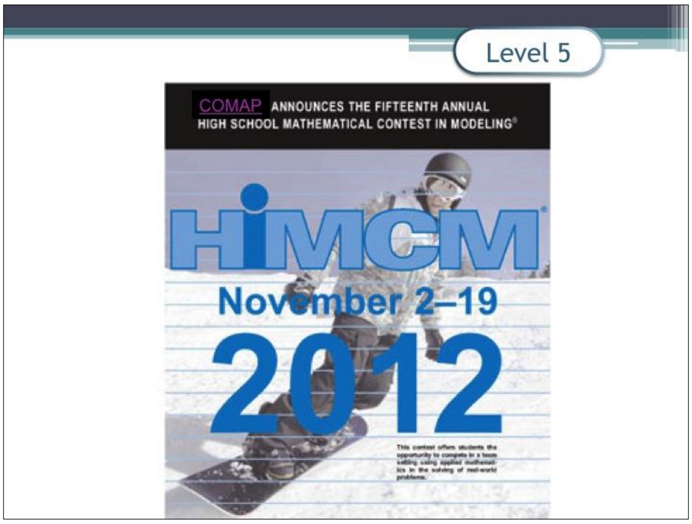
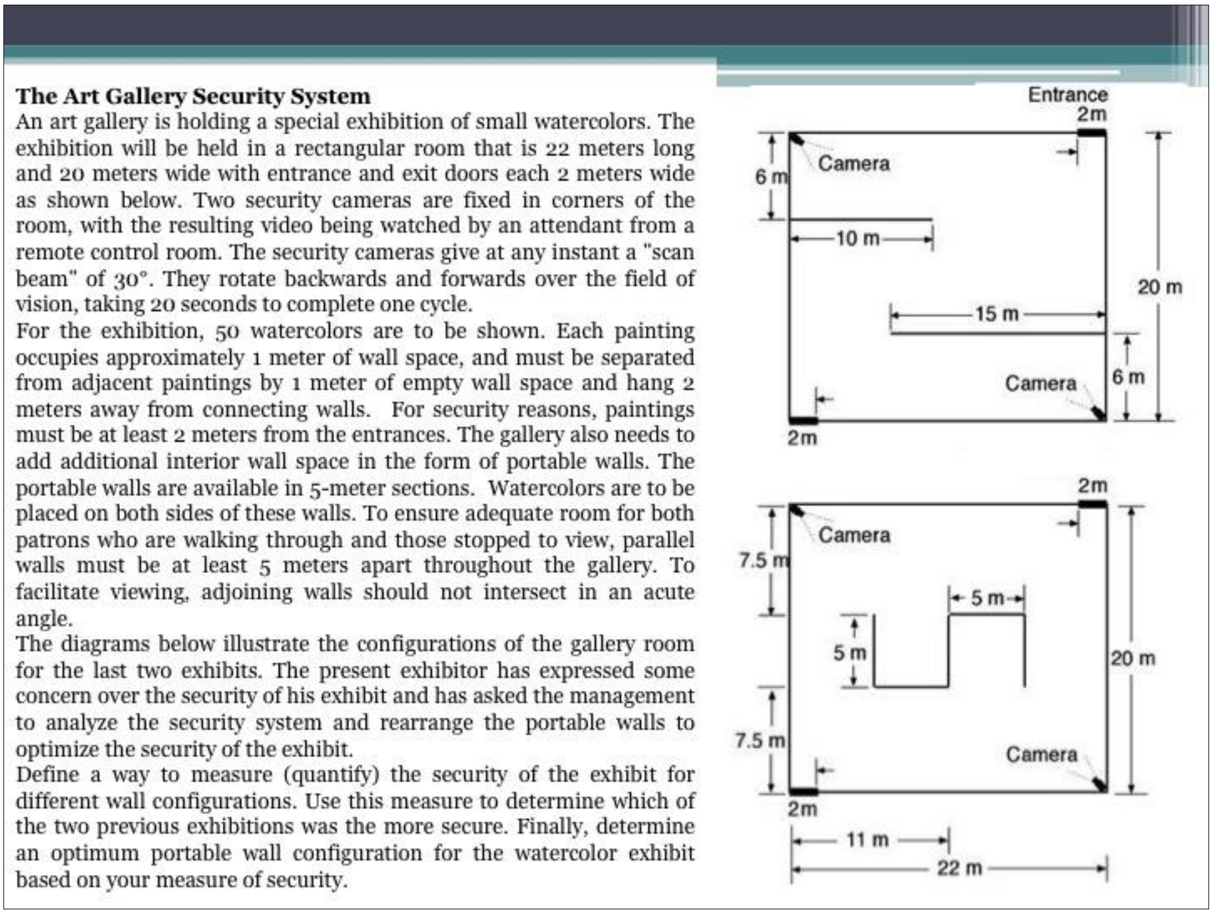


Fig. IV.18: High School Mathematics Contest in Modeling



The Art Gallery Security System

An art gallery is holding a special exhibition of small watercolors. The exhibition will be held in a rectangular room that is 22 meters long and 20 meters wide with entrance and exit doors each 2 meters wide as shown below. Two security cameras are fixed in corners of the room, with the resulting video being watched by an attendant from a remote control room. The security cameras give at any instant a "scan beam" of 30°. They rotate backwards and forwards over the field of vision, taking 20 seconds to complete one cycle.

For the exhibition, 50 watercolors are to be shown. Each painting occupies approximately 1 meter of wall space, and must be separated from adjacent paintings by 1 meter of empty wall space and hang 2 meters away from connecting walls. For security reasons, paintings must be at least 2 meters from the entrances. The gallery also needs to add additional interior wall space in the form of portable walls. The portable walls are available in 5-meter sections. Watercolors are to be placed on both sides of these walls. To ensure adequate room for both patrons who are walking through and those stopped to view, parallel walls must be at least 5 meters apart throughout the gallery. To facilitate viewing, adjoining walls should not intersect in an acute angle.

The diagrams below illustrate the configurations of the gallery room for the last two exhibits. The present exhibitor has expressed some concern over the security of his exhibit and has asked the management to analyze the security system and rearrange the portable walls to optimize the security of the exhibit.

Define a way to measure (quantify) the security of the exhibit for different wall configurations. Use this measure to determine which of the two previous exhibitions was the more secure. Finally, determine an optimum portable wall configuration for the watercolor exhibit based on your measure of security.

Fig. IV.19 : A sample of HiMCM question

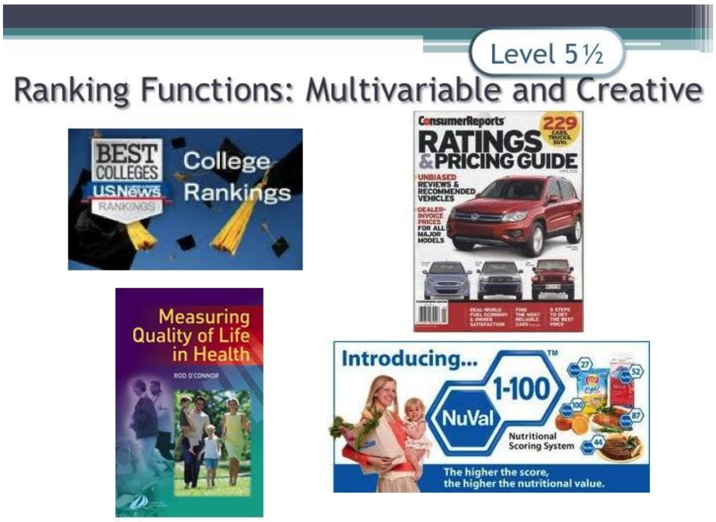


Fig. IV.20: Ranking functions

One of my favourite projects that we do builds on this notion of having to define and measure an outcome and introduces students to multi-dimensional functions that get them beyond two-dimensional graphing. See Fig. IV.20 (from http://news.niagara.edu/nu-again-ranked-among-the-best-by-u-s-news-world-report/ , http://www.amazon.com/ Consumer-Reports-Ratings-Pricing-Guide/dp/B007EZ5K9U, http://www.foodrenegade.com/nuval-nutritional-scores-at-

the-grocery-store/,http://www.amazon.com/Measuring-Quality-Life-Health-1e/dp/0443073198).
Our world is constantly rating and ranking individuals, products, institutions and more. This process is a really a weird one since we are taking variables that do not have anything in common and combining them.

For example, to rate a college, the US News and World Reports looks at test scores, how selective a college is, their endowment, class size, and many other factors. The units for these variables are percents, people, dollars, and more. They have nothing to do with each other but they have to be blended using one function in order to spit out a single score that will permit them to be ranked on a single scale. Many modeling projects require students to develop a way of measuring outcomes and ranking functions are frequently needed. My students have developed functions for deciding which train line had the best service, which college town was the most fun, which food was most nutritious, which dog would make the best pet, and one for assessing the suitability of a tattoo.

A common theme in modeling efforts is that you may not have all of the information that you need and you have to do research to become more knowledgeable. For many situations that we model, it is not possible to get all of the information that we want. We need to learn to work with incomplete information (and/or too many variables with tangled relationships) – consider global warming/climate change. In the United States, we can still have so many climate change doubters because, for them, the idea of uncertainty discredits the results of the modeling process. So teaching students how to do math modeling empowers them to deal with the messiness of the real world. Enabling them to ask questions, and make independent judgments, is a fiercely democratic act that I encourage everyone to work toward.

V. Introducing Mathematical Modeling Skills in the Curriculum

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V. Introduction

At the present time, in Chile we are adjusting our curriculum. We are introducing Mathematical Modeling skills. This APEC workshop is therefore a very good opportunity to know different proposals from different APEC economies and to share experiences in Mathematical Modeling in elementary and secondary schools. I am very grateful for this opportunity to be here and to know what you are doing. Chile is economy with a rather small population and with several educational weaknesses. We do not have the resources to try all sorts of different things; therefore we have to be really careful with what we do and try to learn us much as possible from others. Our mathematics education is not very good, even though it is one of the best ones in Latin America. However, it is not good enough to produce the required amount of high quality professionals, neither to compete successfully with advanced technologies. We have to think carefully about what the challenges for our kids will be in 10 or 20 years from now. What our students are now learning should be very helpful and adapted to the needs in life and work when they will join the workforce. In the Ministry of Education we are now defining the curriculum based on what we predict are the essential skills that will be needed in 20 years from now. One of these skills is mathematical modeling.

IV.2 An USABle Strategy

In order to design strategies to disseminate Mathematical Modeling skills on the school system, we need to understand how scientists, engineers and other professionals do mathematical modeling. Some people build models from zero, but this is very uncommon. Normally, experts have been working for several years on existing types of models. So we need to know the main types of mathematical models that they use. Also there are certain patterns and some kind of techniques that experts use. We need to identify them and then teach that to our teachers.

Following what experts do, we have identified a strategy to learn to do mathematical modeling. We have called the **USAB**le strategy (see also Lingefjard, 2007). It has four stages:

1. **U**se models
   The student first has to learn how to use some models. It could be some very simple models, but powerful enough to be extended later on to important models widely used in certain domains. We have started with first grade students using board games. The student has to follow some rules and then analyze what he obtains.
2. **S**elect models
   From an existing set of models the student has to select the appropriate one for the problem she has to solve. For example, select a board game that best represents a

certain real situation like a war between two armies, or a race competition with obstacles, or real estate market.

3. **A**djust models
   The student has to learn to adjust models by identifying some parameters and carefully changing them according to the data and the problem being solved.
4. **B**uild new models
   Build a completely new model from the existing models is very challenging. Usually engineers and mathematicians work several years building a certain type of models. Most of them are built upon the shoulders of other models.

IV.3 First example

Here is a first example that we have used from first grade. As shown in figure V.1, there is a board with numbers. In each cell, the number represents the amount of food and the challenge is to find the trajectory a bacteria or worm will follow from a specified starting cell. Instructions are given on the rectangles.

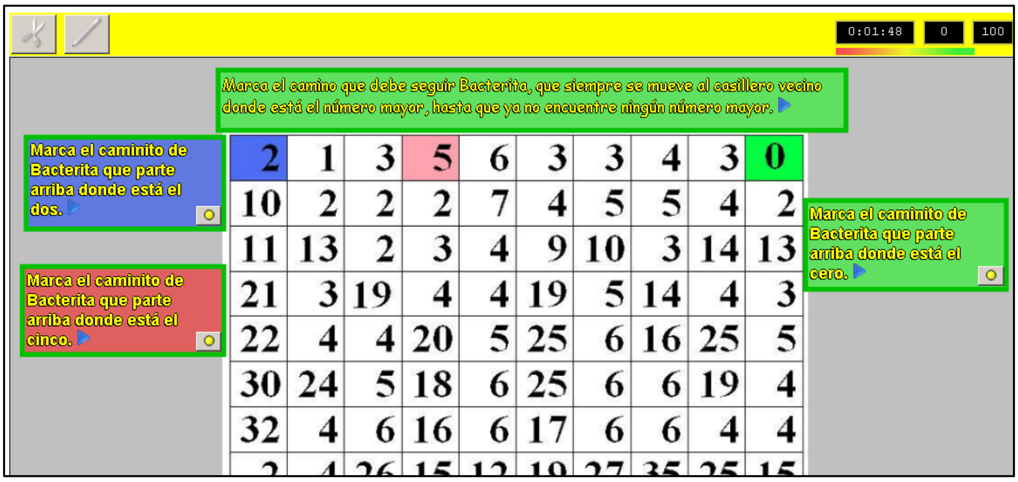


Figure V.1 a: Mark the movement of Bacteria's trail

Figure V.1 b: Three stages on the completion of the task

http://sagde.metaforas.cl/sagde5/desplejercicio?p=/comun_global/Tareas%20Metaforas/01/Num/01-Num-Bacterita-001.cx6

One instruction is in the blue rectangle. It says that the student has to click on the blue cell, which is the starting point for a specific worm. In this case, it happens to have 2 amounts of food. If she cannot read the instructions then she can press a small "play" button and the computer will read aloud the instruction using text to speech facility. Then the student has to choose a next cell and click on it. The selection has to follow two rules: she has to click on a nearby cell (this is defined as a cell that has one common border or common point with the present cell) and the cell has to have the maximum number (amount of food) of all the nearby cells. This means that the bacteria or worm will move to the nearby cell with the most food. The student then continues clicking until there is no cell nearby with higher amount of food. In this way the worm starts at the blue cell with a 2, and the student has to decide to go to the cell with 1, or the one with a 2, or the one with a 10. Here she should select the cell with 10. Then from 10, she has to look around and go to 13, then 21, then 22 and so on. The student continues to choose cells until there is no greater number in its neighborhood. For this game, the basic math skill is to compare whole numbers. The students can do it in their computer or cell phone connected online to internet. This way the teacher can track the progress of the whole class. This board game is part of a very important and powerful kind of models used in math (optimization with gradient type of algorithms), physics (gravitational and electromagnetic potentials) and tropisms in biology (Loeb, 1918).

Next imagine a situation where the student has several boards and has to select the one where the worm will end up in a certain specified position. This is a model selection task. Another model selection activity is where she has to select a board for a different type of real world situation. For example, she has to model how a dog sniffs and moves to find something. In this case the numbers on the cells mean the intensity of the smell at each cell. Here the student has to find that there is a similarity with the worm tropism task and then select the appropriate board. Another example is a modeling task with planets instead of worms or dogs. Here the numbers in the cells are the intensity of the gravitational potential at the different positions produced by a local star. A more challenging task could be to put numbers and their location in the board to model this kind of situation. Also the rules of motion have to be adapted in order to generate trajectories with the shape of the typical orbits of planets around the sun. Thus this type of board games is very interesting not only because it is fun for kids, but because has lots of real world applications. It is a very fertile type of mathematical modeling task. According to (Holland, 1998) board games are as critical to science as numbers "Board games are not usually accorded the same primacy as numbers, but to my mind they are equally important cornerstone in the scientific endeavor".

IV.4 Obstacles and the CCS path

Now let me discuss the obstacles we have found to the introduction of Mathematical Modeling in schools. They are similar to the ones describes by other researchers (Ikeda, 2007). Some of the obstacles are:

1. The typical standardized tests do not include mathematical modeling. Therefore some teachers see mathematical modeling as a waste of precious time instead of teaching something useful to pass the tests.

2. There is none or a very little number of examples of models that teachers know. On the other hand, to create new examples is not easy and needs a lot of time. So we need to have a library of simple examples. They have to have great potential in different domains and also they have to be suitable to create a progression that can be continued throughout grade levels.
3. There is the teacher's belief that this is a waste of time. They believe students should be doing typical math exercises instead. So we have to be able to show to them that this is going to be very useful for them and integrate this vision with their common sense.

A pedagogical strategy to cope these obstacles is called the Concrete → Computer → Symbolic (CCS) path. It is similar to the Concrete→ Pictoric → Symbolic path. In this case the path is:

1. **C**oncrete models.
   We start with a concrete model. Thus modeling does not look frightening. For example, use a known board game such as chest. In this case it models a battle between two armies. These types of models are not difficult for teachers. Normally teachers and students like them. If the board game makes the student to practice other required mathematical skills (like comparing numbers) then it is much better for the teacher. In this case the fear of wasting time disappears since those other skills are required on the standard tests.
2. **C**omputational models.
   After the concrete model becomes familiar, start using a computer model. For example, continue with a computer game similar to the board game. From the point of view of the students, computer games are very attractive. They really love them. From the point of view of the teacher, the computer helps to monitor the progress of each kid. Teachers appreciate any tool that makes it easier to track students learning. On the other hand, games are some kind of simulations (Honey & Hilton, 2010), and running simulations is exactly a typical activity that experts do in building mathematical models.
3. **S**ymbolic models.
   After having played or simulated a lot of times, students can realize that certain patterns emerge. Then they can try to prove that these patterns always hold or otherwise find counterexamples. The symbolic model is a powerful notation and framework to do this task.

IV.5 More examples

Here is another example we have used from first grade. This is a model for a critical and core concept in science education: natural selection. Students hold in their hand binder clips. The binder clip represents the beak of a bird. There are big and small binder clips hat represent birds with big or small beaks. Figure 2 shows third grade students playing in the classroom floor. Each one tries to grasp a ball with his binder clip. The balls represent food. Once a ball is grasped it is taken out of the floor, since it is assumed that is being eaten. There are small and big balls. The binder clip that grasps a ball will reproduce and have descendants in the next generation. The students play and have to predict for each generation the distribution of clips size using graphs as shown in figure V.2.

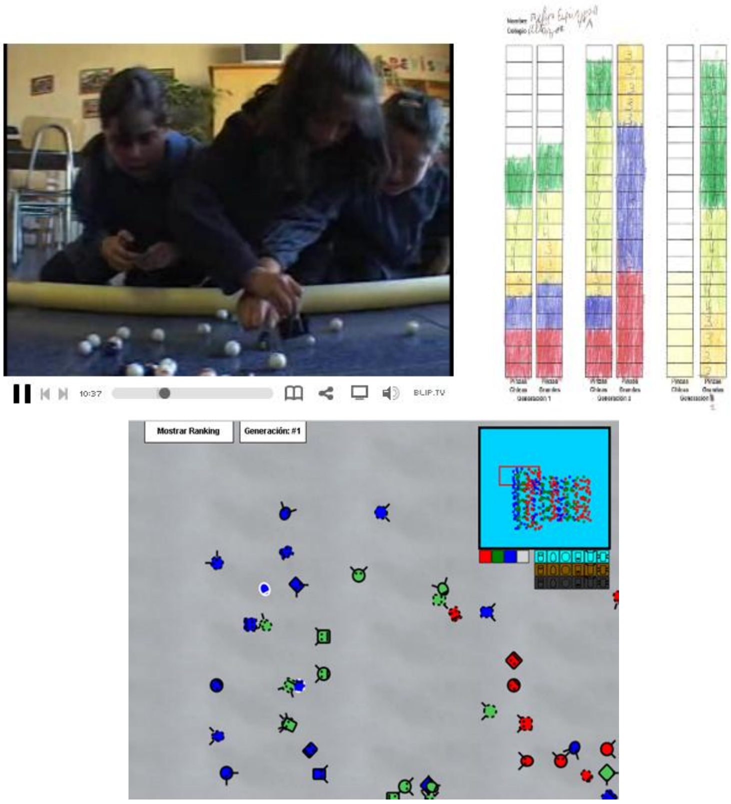


Figure V.2: Natural selection game with binder clips, predictions using graphs and an online version

http://organismosmagicos.metaforas.cl/log/player/7183/1

There is a computer version of this game with three species of animals and six genes or features for each species. The students have to predict the distribution on the population of the six features and the size of the population of the three different species.

Now we show a different example that we have used only with high school students. It is a catapult tournament. Using an instruction manual and construction scaffolds (see figure V.3), students build a trebuchet catapult. The construction takes about one hour. Then they have to identify key parameters to shoot soccer balls as far as possible or to shoot a target. They need to design experiments, control variables, collect data, and then do some modeling in a spreadsheet in order to be able to win in the shooting tournament. We have done some seminars for teachers to do the whole process and analyze the mathematic modeling behind it.

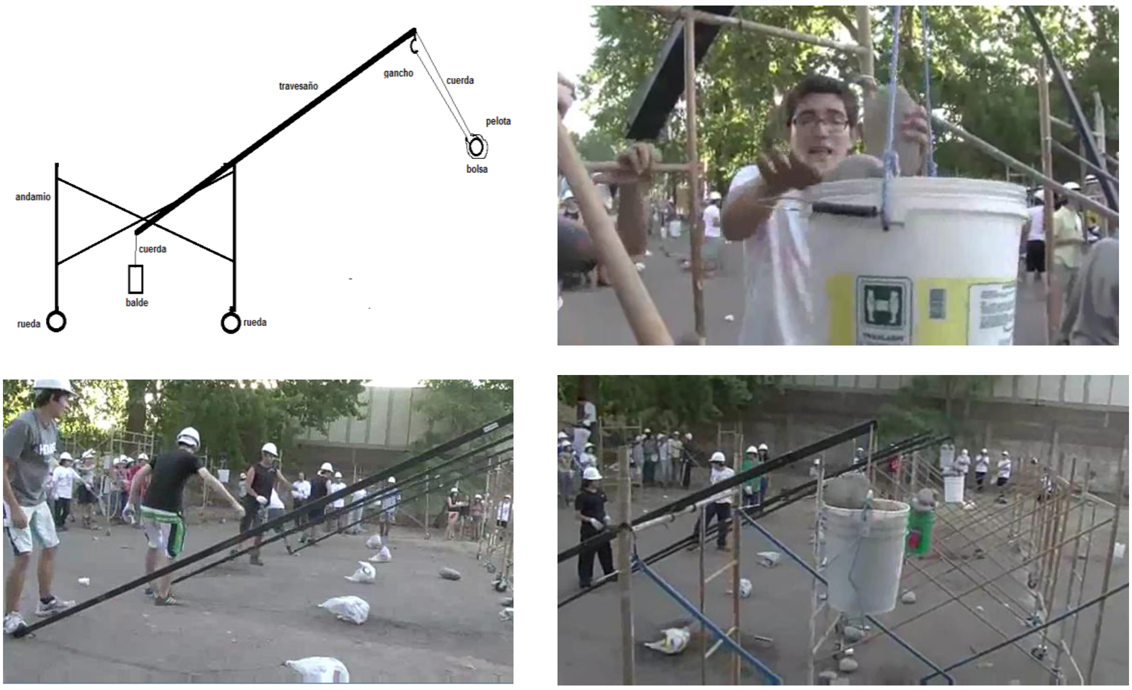


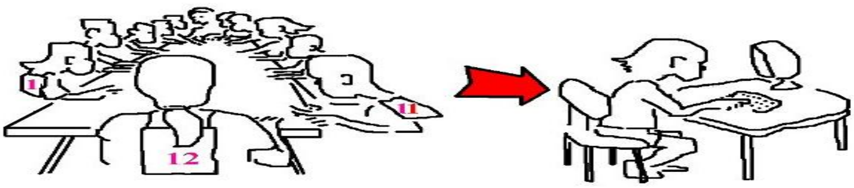
Figure V.3: Building trebuchet catapults and doing math modeling with high school students

We have reviewed three concrete games, the associated computer models, and some kind of more abstract modeling. But, why games? First, because play it is a natural mechanism for learning (Pellegrini, 2009). It is not only used by humans. For example, animals like dogs and cats learn how to hunt by playing (Burghardt, 2005). It is natural pedagogy. Second, games like board games have explicit rules that the student has to follow. This is critical. We are making the student to practice the ability to use a model. It is not necessary to be a small board. It could be a very large as the one shown in figure V.4. Students have to learn to follow the rules. Interestingly, after a while, student will want to invent their own rules. This is an excellent opportunity to challenge them to produce no contradictory rules, rules that at any state will give instructions what to do, and particularly rules that somehow captures certain real world situation.

Figure V.4: Board game for kindergarten kids

Another type of fertile models is the classification type of models. Here the student has to discover a pattern. The simplest case is the one when she has to predict one out of two possible outputs. For example, if someone is lying to you or not, or if someone is sick or not. Normally you have a history of similar cases where you can see if there is pattern that can be helpful in the present situation. This problem is represented in a game where you have to predict the color of a cell that in inside a box. You can examine opened boxes that were randomly selected on previous rounds. With this history you can detect if certain features of the box are related to the color of the cell, and then make your decision. This kind of decision making process is very important in real life. Figure V.5a shows the set of all boxes, figure V.5b shows when a box is selected, Figure V.5c shows the measurements of the different characteristic of the selected box, figure 5d shows what was inside (it has 12 cells). Figure V.5e shows the team members, each one with a number and is in charge of the predictions to the corresponding cell, and figure V.5f shows the process of submitting the bets to a computer version of the game.

(a) (b) (c) (d)



(e) (f)

Figure V.5: Basic decision modeling

In this game (Araya, 2007; 2011, 2012), called "Magic Surprises", you get two points if the prediction is correct, and minus one if it is incorrect. You are allowed to say "I don't know", but you get only one point on that case. You can also submit a rule (a model), as the one shown in figure V.6. If the model makes a correct prediction you get 4 points. If it is incorrect you lose 4 points.

IF

length **+ 2.5 x** ***width*** **> 3**

THEN

cell color **= white**

ELSE

cell color **= black.**

Figure V.6: Rule constructed by student and submitted during the game

This game is a good model of several real world situations. For example, credit risk in banking. In this case the box represents a client and the features of the clients are gender, salary, how long he has been working on his job, education, etc. A black or white cell is interpreted as the future behavior of the client: if he pays back the credit or not. Another real world situation is medical diagnosis. In this case you have to decide if the person is sick or not according to symptoms and the history of several previous cases. The box is a patient, and the features of the box are the different patient symptoms. But for kids, these types of real world problems are not always very exciting. However an application to soccer or another sport is almost always very interesting for them. In this case the box represents two teams that are going to play a soccer game. Each team has its own features. For example, the characteristics of the Chilean team are confronted to the ones from the Indonesian team. Typical variables are the number of goals per game, the ranking in the soccer association, and other performance indicators. These indicators are computed from the selection tournament previous to the world cup. The student has to combine them and propose a formula to decide automatically the potential winner. So before the last soccer world cup, we gave all the data of the previous soccer world cups, the one in Germany, and the one in Japan and Korea, so that students could test the prediction capabilities of their models before submitting them. In comparison to predicting credit risk and medical diagnosis, predicting soccer games was much more engaging.

In Chile, we have a tournament of this type of games. See figure V.7 to see the number of viewers who access the online game from Jan 31, 2009 to Aug 16, 2010. Before the tournament happens, data shows they were playing together at the same time. Normally this is Friday in the morning. So at certain time, teams over 100 schools, play at the same time. So this is like the preparation before the real tournament.

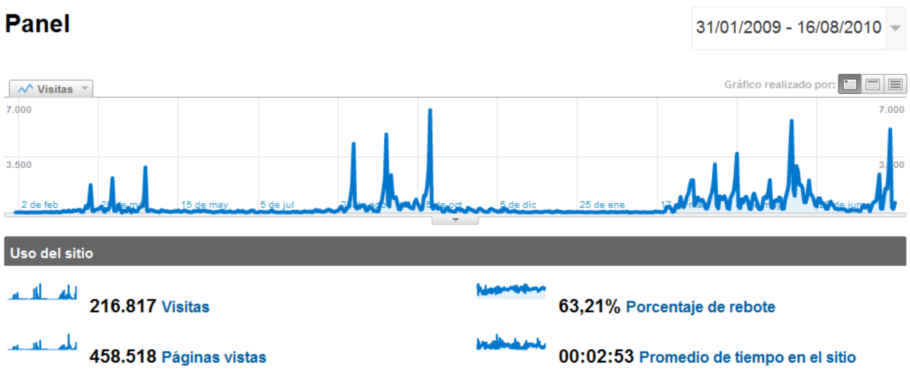


Figure V.7: the number of viewers of the online game during 2009 and 2010

Because all students are playing in the computers, and the movements are saved, we can analyze the data to search how they are playing. For example, figure V.8 shows the scores in the game Magic Surprises. Using cluster analysis we detected three clusters of students. On each graph are plotted for each round the percentages of students that obtained -4 points (yellow), -1 points (purple), 1 point (crème), 2 points (green) and 4 points (blue). The graph in the left shows the behavior across rounds of the first cluster. Some of them were improving and somehow detecting the hidden patterns. However, these students didn´t use models. The graph in the middle shows the conservative players who answer “I don’t know” most of the time. The graph on the right is the cluster of students that were able to build an explicit model and submit it. Instead of betting directly, using what would be an implicit model (Epstein, 2008), they send a program (rule) and the program computes the answer. They were generally successful in predicting the outcome. Roughly 10,000 students from third to 10th grade played the game on the tournament and only less than a third of them built models while playing in the tournament.

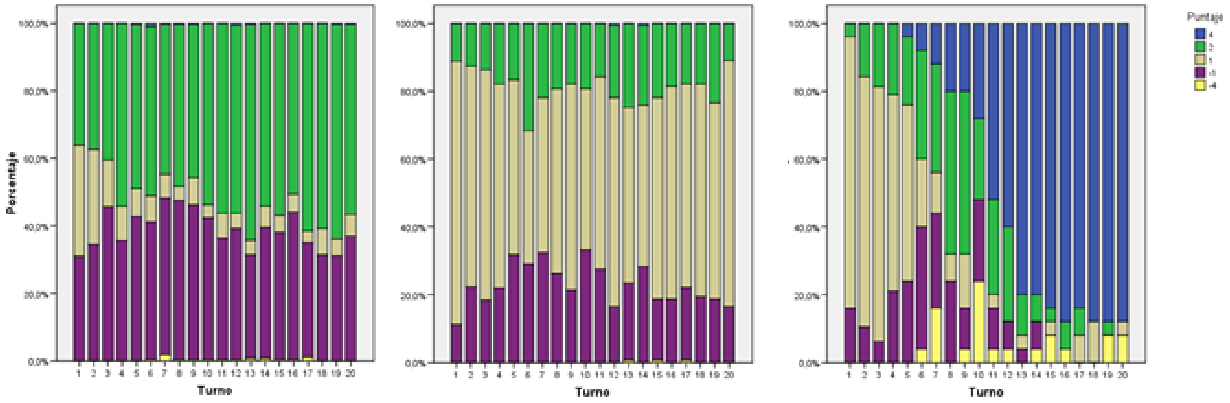


Figure V.8: Three clusters of students in the Magic Surprises game

IV.6 State space models

One widely used type of models is the one that captures the evolution of a system like a tank that accumulates water. These are the state space models, like the following one

For example, in a tank with water, x(t) is the amount of water at time t. The equation gives the dynamics of the amount of water. That is, how the amount of water increases or decreases from one time to the next one. More specifically, the equation says that the amount of water in time t+1 is a fraction of the amount at time t plus some input that is being introduced with water from a hose. This kind of modeling uses a very simple math that it can be easily simulated in a spreadsheet.

Here is another situation modeled by this type of equations. Now x(t) is the emotional state of a person at time t. This is a number between -1 (very depressed state) to 1 (very happy). The equation is

This is a simplified version of the models in (Gottman et Al, 2005). In the equilibrium x = ax + b, and therefore x = b/(1-a) . This means that the emotional state in the equilibrium is around this level. If a is less than one and if the person is somehow moved out of this level his emotional state will recover its equilibrium with a velocity that depends on a. For example, if you win a lottery you will be very happy but after some time you will reach your typical mood. The level of the equilibrium state and the velocity of recovering that equilibrium state depends on a and b. These are personal parameters. Each person has its own. All of these can be easily simulated on a spreadsheet.

Now let us consider the case of a sentimental relation in a couple. In this case there are interactions between them that impact their mood states. Let M(t) and F(t) be the respective emotional states of the male and female at time t. The value of M and F can go from minus 1 to plus 1. As before, one means feeling very good and minus one means feeling depressive at that moment. Consider

If there is no F(t) in the first equation, then the equation will be . As in the previous case when we analyzed an isolated person, the emotional state in the next step M(t+1) is depended on the emotional state at time t and the values of parameters a and c. The dynamics of the emotional states of the female is similar, but with different parameters.

When we put both persons together, each one affects the other. The male is affected by the female and vice versa. The strength of the interaction is expressed in the coefficients e and f. In figure V.9 two spreadsheet simulations are shown. On the left graph there is a couple in constant oscillations of their emotional states until reaching a neutral state for both persons. On the right graph, there is another couple where both rapidly reach positive emotional states even though they are different. One has a higher emotional state than the other. It would be interesting to analyze if the rapid increase of mood out of a very depressed emotional state is due or not to the interaction in the couple. What is very important to highlight here is that now that we have a model, there is certain data that will be very interesting to collect. This guiding process is one of the main goals on model building (Epstein, 2008).

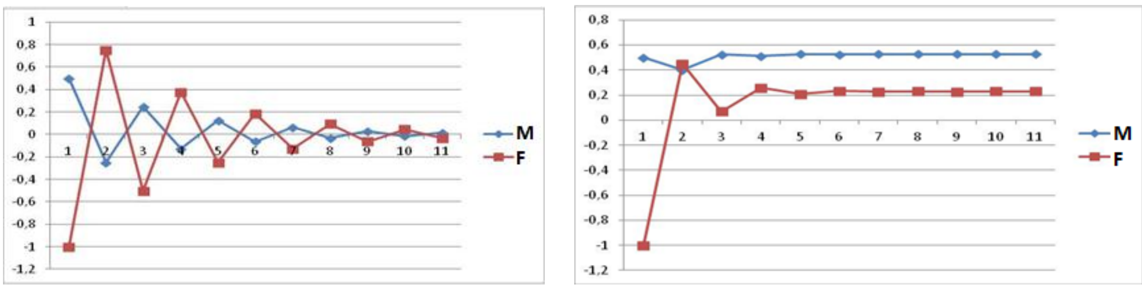


Figure V.9: Two cases of male and female evolution of emotional states in 11 time steps

These types of models are very attractive to adolescents. They come to see that mathematics can be used to understand not only problems with stones and buildings, but also to understand emotional and social phenomena.

IV.7 Conclusions

So to summarize, we have seen two strategies for introducing mathematical modeling. The USAB (Use, Select, Adjust, Build) and CSS (Concrete, Computer, Symbolic) strategies. First, we teach the kids to learn certain models, hopefully models which are powerful and fertile, like the one based on a potential function that is used in physics and in biological tropisms. Or the model that is based on the pattern detection and decision game that is used for risk analysis, diagnosis and all sorts of decision situations.

The second step is to select the model. Once you know many kinds of model, you need to select a model which best suit to the situation and test it. The third step is to adjust some parameters. As it is shown in the models shown, we need to adjust the values of parameters based on real data and all kind of graphs. And the final step is to build some relatively new model. However, this is very challenging because usually mathematicians, engineers and economist work several years using well known models before introducing important modifications or creating radically new models.

The second strategy is to start with concrete models. This way the student can better understand what is going on. It will also be more engaging and less frightening. It could be just a board game or a physical model. This is also what usually engineers do to understand a phenomenon and to explore how their solutions will work. If they want to understand how a river, floods and winds impacts on a bridge that they are designing, engineers make a scaled model from the real one. In the second stage, we propose to build computer models and run simulations. In the third stage we propose to analyze the symbolic equations. For example, in the model for the emotional states of a couple the analyses of the equilibrium states requires solving a two by two linear system. This is a typical problem for grade 10 or 11, but here applied to understand human emotional states.

Figure V.10 shows a matrix that summarizes both strategies, and how each activity can be classified.

	Concrete	Computer	Symbolic
Use			
Select			
Adjust			
Build			

Figure V.10: Matrix of USAB and CCS strategies

What is needed to introduce math modeling in the school?

1. First, we need more examples. One of the objectives to participate in workshops like this one is to get good ideas for mathematical modeling in elementary, middle and high school. It would be great to have a database of math models from all over the world that everybody can access. We need powerful models that can be used in several kinds of situations and across several grade levels and applications. It would be very useful to start a network of practitioners and create mechanism to assess the potential of the models proposed.
2. Second, we need to know how to assess modeling skills in test at economy's level. One possibility is the use of the computer. Nowadays students use computer to play games and the games track the user´s progress. But governments still want to have regular paper test like PISA or TIMSS. So we need some ideas here.
3. Third, we need to know how to train teachers on modeling skills. This is a real challenge since it is completely new for them. This will take a lot of time and teachers would normally say "why should I do this? I have other stuff to teach". We

have tried online game tournaments with hundreds of school participating. The games are really math modeling tasks. In order to prepare the student teams, we have had online seminars to train teachers on how to do modeling and how to advise students to play with good strategies.

4. Fourth, we need to know how to implement massively the introduction of modeling skills
   We have done this in small scale, meaning thousand students per year. We need to plan this really massively for the whole economy.

According to Epstein (2008) we are always constantly modeling, but the models are implicit. "The choice, then, is not whether to build models; it's whether to build explicit ones. In explicit models, assumptions are laid out in detail, so we can study exactly what they entail". Mathematical modeling is what building explicit model is about. It is a key thinking skill that all our students should learn.

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