Asia-Pacific
Economic Cooperation

A Progress Report:
A Collaborative Study on Innovations for Teaching and Learning Mathematics in Different Cultures among the APEC Member Economies

International Symposium “Innovation and Good Practices for Teaching and Learning Mathematics through Lesson Study”
June 14 – 17, 2006, Thailand

APEC Human Resources Development Working Group

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By Center for Research in Mathematics Education
Faculty of Education, Khon Kaen University
123 Khon Kaen University Mittraphap Highway
Khonkaen City 40002
Thailand

Produced for
APEC Secretariat
35 Heng Mui Keng Terrace Singapore 119616
Tel: (65) 67756012 Fax: (65) 67756013
Email: info@apec.org Website: www.apec.org

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PREFACE

This is the progress report on the APEC project ***"A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies".*** It included the result of APEC – KHON KAEN International Symposium.

At the third APEC Education Ministerial Meeting held on 29-30 April 2004 in Santiago, the ministers defined four priority areas for future network activities. "Stimulating Learning in Mathematics and Science" is one of the four priority areas. Based on this priority, the project "A Collaborative study on innovations for teaching and learning mathematics in different cultures among the APEC Member Economies" was approved by APEC Member Economies in August 2005. The project is managed by the Center for Research on International Cooperation in Educational Development (CRICED) in the University of Tsukuba and the Center for Research in Mathematics Education (CRME) in Khon Kaen University. There are four phases in this project:

In phase I, an open symposium and closed workshop among key mathematics educators from the cosponsoring APEC Member Economies was held during 15-20 January 2006 in Tokyo by the University of Tsukuba under the management of the Center for Research on International Cooperation for Educational Development (CRICED). The purpose was to further develop a research proposal and collaborative framework for the development of innovation and good practices for teaching and learning mathematics. As the results "Lesson Study" is the key innovation of the teaching professional development to develop innovation and good practices for teaching and learning mathematics.

In phase II, based on the agreed collaborative framework, each cosponsoring APEC Economy conducted the research in the real classroom setting in each his/her home country to develop innovation and good practices in teaching and learning mathematics through lesson study during February – May, 2006.

In phase III, an APEC International Symposium on "Innovation and Good Practices for Teaching and Learning Mathematics through Lesson Study" was organized in order to share and reflect on each Economy's research results and good practices based on the developed framework of Tokyo meetings. The Symposium was hosted by Khon Kaen University, Thailand on June 14-17, 2006. Based on the financial support of APEC project, the Office of the Commission on Higher Education, and Khon Kaen University, the project aimed at organizing:

- APEC Open symposium: "Innovation and Good Practice for Teaching and Learning Mathematics through Lesson Study, Khon Kaen Session"
- APEC Specialist session: "Presentation on good practices of teaching and learning mathematics through Lesson Study".

245 participants and observers attended this meeting. 200 local participants and observers including university lecturers, mathematics teachers, experts and educational policy makers related to mathematics education in Thailand and 45 participants and observers from 13 members of economies of APEC including Australia, Chile, China, Japan, Hong Kong, Indonesia, Laos PDR., Malaysia, the Philippines, Singapore, South Africa, USA, and Vietnam attended this meeting.

In phase IV, the ‘APEC Workshop on: Improving the quality of the mathematics lesson through Lesson Study’ was held in Thailand in 24-27 August 2006. In this workshop, Japanese teaching method was proposed to teachers of Thailand in the style of workshop on Lesson Study. Teachers who belong to Attached Elementary School of University of Tsukuba, Japan came to Khon Kaen to demonstrate two phases of Lesson Study – teaching Thai students in the real classroom and reflecting on teaching with Thai teachers. Activities in this phase really reflect the title of the project. In addition, these activities are also effective to support the movement which is developed in northeast area of Thailand by Khon Kaen University.

For the most benefit of APEC member economies to share their knowledge on lesson study, based on the results of these APEC - Khon Kaen International Symposium, we will publish a book consisting of reports and VTRs of Lesson Studies from participating economies.

We appreciate the Office of Commission of Higher Education, Ministry of Education and Khon Kaen University for their fully support the APEC project “A Collaborative Study on Innovations for Teaching and Learning Mathematics in Different Cultures among the APEC Member Economies.” More importantly, we would like to thank all members of CRME and staff of Faculty of Education for their contributions to organize the symposium and to complete this progress report. At last, I would like to use this space to give my gratitude to Prof. Dr. Alan J. Bishop of Monash University, our keynote speaker, for his great contribution to mathematics education community in the Great Mekhong Sub-region countries in the donation of a complete set of Educational Studies in Mathematics (ESM) to house in the library of Khon Kaen University.

October, 2006
APEC Project Overseers
Suladda Loipha and Maitree Inprasitha (Khon Kaen University, Thailand)
Masami Isoda and Shizumi Shimizu (University of Tsukuba, Japan)

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Opening Remark

Dr. Suchart Muangkaew

Deputy Permanent Secretary, the Office of the Commission on Higher Education

It's an honor for me to preside over this International Symposium. From the report, I learned that in these symposium mathematics researchers from 12 APEC member economies have been working together since April 2004. This kind of long term study will ensure promising outcomes. In the end it should yield excellent research results in mathematics education for stimulating mathematics and science learning in all member countries. I believe that this type of collaborative research should lead to a strong commitment for academic exchange especially in the area of mathematics educational research.

The 21st Century is "the century of knowledge-based societies" in which we consider knowledge and wisdom as key to success and development. The theme of this symposium "International Symposium on Innovative Teaching Mathematics through Lesson Study" indicates the driving force of researchers to innovate their teaching and learning approaches in mathematics.

I am grateful for the main supporting agency APEC Human Resources Development Working Group and the various institutes which have been fully involved in this event, namely Center for Research on International cooperation in Educational Development in the University of Tsukuba, Japan, the commission of Higher Education Thailand and the Faculty of Education at Khon Kaen University. All of their contributions and efforts are keys to the sustainable progress of this project. Thai mathematics educators from all over the country have the opportunity to participate in this symposium which is a great benefit for Thailand.

On Behalf of the host country, Thailand, I wish this symposium a great success and the continuation of the project for the greater benefit of every member economy countries.

I may now declare the opening of the International Symposium on Innovative Teaching Mathematics through Lesson Study.

Welcome Address

Prof. Dr. Chira Hongladarom
Lead Shepherd of APEC HRD Working Group

As Lead Shepherd, I am pleased to congratulate to all members who took part in the fourth workshop in Khon Kaen during 14th – 17th June 2006 on "A Collaborative Study on Innovations for Teaching and Learning Mathematics through Lesson Study among the APEC member economies".

When the project was proposed during the 27^{th} APEC HRD Working Group meeting, I was very happy that such innovative ideas were being implemented in the APEC setting.

Someone said "I like Mathematics because Mathematics is romantic language" that I agreed with him and I try to promote mathematics for everyone.

Beside free trade and investment among APEC economies, I believe that human resources are a crucial component in bridging the development gap among APEC economies. Therefore mathematics is important not only for mathematics sake but for economic and social benefit. With increasing mathematics skills the APEC community will develop systematic thinking and will benefit from the rising trend of knowledge based society. Among the 13 APEC member economies, we shared different approaches in teaching mathematics to disseminate best practices.

I look forward to witness the phase of the project in Khon Kaen, I will try to inform the public at large about the benefits of such projects through my television program.
Finally, I would like to express my thanks to Prof. Masami Isoda from the University of Tsukuba in Japan and Assoc. Prof. Dr. Suladda Loipha from Khon Kaen University.

Thank You

Welcome Speech

Prof. Dr. Sumon Sakolchai

President of Khon Kaen University

I am delighted to welcome all participants. On behalf of Khon Kaen University, I would like to express my sincere gratitude to the Lead shepherd of the APEC Human Resource Development Working Group, Prof. Dr. Chira Hongladarom whose support has made this research project possible. I also would like to extend my deep gratitude to all guest speakers and participants who came a long way to participate in this symposium.

This symposium is a product of collaboration between the University of Tsukuba, the Commission of Higher Education under the Ministry of Education in Thailand, Khon Kaen University and researchers from APEC member countries.

This symposium will provide opportunities for participants to address their issues, share experiences, expand collaborations and partnerships in mathematics research and make contacts with outstanding mathematics educators.

The organizers will ensure opportunities for participants to meet and discuss closely outside the formal sessions. I believe that valuable idea and strategies based on innovation implemented in the classroom settings and working experiences will be revealed and distributed throughout this meeting.

Based on findings from the past phases of this research project, it is believed that the hard work of APEC members in teaching and learning innovation will contribute to the development of mathematics education in all participant countries. Moreover, the project will promote large scale collaborative international research and ensure the continuity of international exchange activities for years to come.

Once again, I would like to welcome you all to Khon Kaen City and Khon Kaen University and would like to express my gratitude to all supporting institutes and organizing staff. Thank you very much.

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APEC INTERNATIONAL SYMPOSIUM

INNOVATION AND GOOD PRACTICES FOR TEACHING AND LEARNING MATHEMATICS THROUGH LESSON STUDY

MATHEMATICS EDUCATION FOR THE KNOWLEDGE - BASED SOCIETY

Alan J. Bishop
Monash University
Melbourne, Australia

1. What am I offering in this address?

It is a great honour for me to give the opening address to this conference and of course I am very happy to be here again in Khon Kaen, Thailand. I am also happy with the topic which I have been given by the organisers, and my talk today will offer the following five contexts for you, which I will briefly clarify now:

- A frame for the conference discussions?

This conference is focussed on teacher education in mathematics and particularly on the use of 'lesson study' as a means for developing both the theory and the practice of mathematics teacher education. But it is necessary to keep this topic framed, particularly in such a short conference as this is, in order that we maximise our time together.

- A context for considering generalisations?

Mathematicians and mathematics educators love generalising – it is valued as one of the basic means for developing mathematical ideas. The challenge for us however is that where mathematics seeks to develop ever more abstract ideas, teacher education must always strike a balance between abstract theory and concrete practice.
Both student teachers and experienced teachers will reject any ideas for teacher education that do not strike what they feel is the right balance between the two objectives.

- An explicitation of some hidden assumptions?

In my research on values in mathematics education, it is clear that most values teaching and learning takes place implicitly in the mathematics classroom. This is also likely to be the case in the context of this project, which is even more problematic since we come from very different cultural and social contexts. It is vital that in our discussions we keep aware of the hidden assumptions and values which are not necessarily shared by all.

- A personal view on the values involved in this project?

Having mentioned values above, it is necessary for me also to clarify my values and assumptions within this conference topic. No researcher is value-free!

- An opening up of some of the issues involved?

Although my topic is not especially about lesson study, nevertheless I feel it is necessary for me to at least expose my ideas about some of the issues involved in this development. (I must also ensure of course that you do not go to sleep!)

2. Definition of knowledge-based society

My topic is certainly an interesting one, full of issues of definition, values, goals and predictions. But in 2003 there took place the World Science Forum in Budapest, Hungary, and their theme for that conference was Knowledge and Society (see website ref.) In it they gave a useful definition of a Knowledge-based society, and here are the main points:

- A knowledge-based society is an innovative and life-long learning society
- It possesses a community of scholars, researchers, technicians, and firms engaged in research and in production of high-technology goods and service provision.
- It forms a national innovation-production system, which is integrated into international networks of knowledge production, diffusion, utilization, and protection.
- Its communication and information technological tools make vast amounts of human knowledge easily accessible.
- Knowledge is used to empower and enrich people culturally and materially, and to build a sustainable society.
- Innovative
- Life-long learning
- National and international networks of learning communities
- ICT goods and service provision
- Empowerment/enrichment of society culturally and materially
- A sustainable society

In some ways this is a formidable list, containing both descriptive and prescriptive ideas. Every country would have something to aspire to from this list and all of us attending this conference here today would have reservations about whether our countries are achieving any of these goal descriptions. But it is good to have such a challenging list to begin our deliberations here.

3. How to consider education in this new context?

In particular it is a challenge to consider education within this new context. But is a knowledge-based society really a new idea? We should ask ourselves what is different now. Society has always used and taught knowledge, but originally it was the family context which provided the education, from whom the knowledge came and with the elders being the 'teachers'. Gradually as education became more formalised, the schools developed from the families. Also the content of what was taught became more organised, and became based on the knowledge supplied from the 'academy'. Finally the teachers became officially recognised, needing official qualifications and eventually being specifically trained.

Now as the knowledge society is developing, we find that the new knowledge comes from 'outside' the accepted sources, from the Web, from the media, from peer-group networks and also from wide international sources. But many questions also arise for us in education: Whose knowledge is it? Who is producing it? Whose personal knowledge is being exploited and whose personal knowledge is being ignored? Basically the question now facing us is: What is the source of the authority of any new knowledge?

4. Kinds of education => Kinds of mathematics education

Coombs(1985) gave a very helpful analysis in his book ‘The world crisis in education.’ He based his analysis on three kinds of education: formal, non-formal and informal. According to Coombs, there are crucial distinctions to be made between these, and I feel that we too need to be aware of these within our special field. Thus I offer you three kinds of mathematics education whose distinctions are I think crucial in considering our roles in a knowledge-based society. The three sets of characteristics are based on Coombs.

Formal mathematics education is the formal system most of us are part of, and it consists basically of the state system which exists in most countries. It is largely the only kind which gets recognised in research in our field, and operates up to student ages of around 16 or 18 years. It is

- Structured
- Compulsory
- A coordinated system, which is
- Staffed by recognised teachers

Non-Formal mathematics education is the kind of non-compulsory and optional education offered by courses such as for adult education, or vocational education and training. For formal school-age students, it could be after school classes, cram-school classes etc. Generally it is:

- Structured
- Non-compulsory/optional
- With a specific focus
- Coordinated to a certain extent, and
- Some teachers are recognised, some not.

Informal mathematics education is the largely unstructured and often accidental education which comes from a variety of sources, and ‘happens’ to all of us. Whether it is on the Web, on TV, via computer programs, in the papers, or journals, or occurring at conferences like this one. Its characteristics are that it is:

- Unstructured
- Accidental
- Uncoordinated, and with largely
- Unrecognised ‘teachers’

Coombs particular contribution for me was that we have to consider the last category as a form of education, to look at it through educational eyes. It makes us think about questions like Who are the ‘teachers’? What is their agenda? What is the nature of the mathematics being taught? How do these ideas intersect with those being taught in the Formal system?

5. Where is development happening?

If we continue with these three categories, we can ask some more interesting questions, such as where is development happening in mathematics education? Regarding the three categories, we can summarise things this way:

Formal Mathematics Education (FME):

- Developing slowly in terms of mathematical knowledge
- Developing slowly in terms of pedagogy
- Difficult to change the system
- Difficult to change the examinations
- Student input to changes limited

Non-Formal Mathematics Education (NFME):

- More responsive to knowledge changes
- Pedagogical developments less restricted
- More scope for individual teachers to develop courses and materials
- Less controlled by examinations
- More responsive to student inputs as ‘clients’.

In-Formal Mathematics Education (IFME):

- Responsive to, and often initiating, knowledge changes
- Opportunistic with respect to ‘pedagogical’ changes, no examinations
- No formal teachers means greater experimentation and innovation
- Client-led learning
- Lack of control on authority for knowledge

6. Responses of Mathematics Education to the growth of the knowledge-based society

Now we can begin to identify how mathematics education is responding to the growth of the new knowledge based society. For example we can see that IFME is highly responsive, and is often leading the developments. Via the Web, new computer programs, and international networks, we are seeing many developments (or pressures for developments) taking place.

NFME is responding in some ways, in particular in changing the structured courses to respond to client needs in the training and vocational education sectors. In fact as the business models for the NFME providers become much more sophisticated, and in line with other businesses, this sector of mathematics education is exerting much influence on the formal sector. In some ways the borders between IFME and NFME are becoming rather blurred.

On the other hand, and in stark contrast, the FME sector is slow to respond, and even then with minimal changes. There are some changes in curriculum taking place, particularly with the pressures from those who are advocating more emphasis on Numeracy, but there have been few changes in pedagogy, even though ICT is becoming more prevalent in schools and classrooms.

7. What particular developments should we aim for in FME to prepare our students for the Knowledge-based society?

Firstly any Formal Mathematics Education must balance several complementarities:

- Individual growth v. class/group/grade development
- Traditional content v. expanded knowledge
- Traditional pedagogy v. ICT and student-led pedagogical approaches
- Formal systemic examinations v. individual assessment

So bearing these balances in mind, let us explore the definitions of, and criteria for, a knowledge-based society and see how we would develop our FME in our different countries:

Innovative society

- Teaching should encourage more creativity in the students
- Individuals' and groups' original ideas should be valued by teachers
- Assignments should allow creative initiatives
- Assessments should reward creative ideas and solutions to mathematical problems

Life-long learning

- Laying the skill foundations for problem-solving and creativity
- Teaching information searching
- Teaching information validating
- Developing publication and knowledge-sharing skills

National and international networks of learning communities

- Encouraging knowledge networking
- Demonstrating learning community activities
- Contributing to, and using information from, those communities

ICT goods and service provision

- Increasing the familiarity of teachers and students with ICT equipment and software
- Recognising the limitations of ICT as information and communications media

Empowerment/enrichment of society culturally and materially

- Recognising the cultural and historical nature of mathematics knowledge
- Recognising how mathematics assists, informs, and thereby 'formats' society
- Recognising the limitations of mathematical knowledge

A sustainable society

- Mathematics education should embrace environmental education
- Values education should be more explicit
- Balancing individual goals and societal goals should be addressed

8. Final thoughts

Lesson study needs recognising as a socially situated research practice

This is where the Social dimension of mathematics education needs greater recognition (Bishop, 1991). It operates at these five main levels:

Cultural level - language, values, culture, history
Societal level – politics of society, educational institutions,
Institutional level – within institutional rules and goals, internal politics
Pedagogical level – within the classroom, teacher and students as social group
Individual level – individual students' and teachers' backgrounds and goals

Any lesson study research is therefore situated within any particular cultural, societal, and institutional context.

The cultures and values of researchers need recognising

Related to the points above, we should note that no research is ever value free, there are always goals, assumptions, histories and institutional politics at work. Moreover, we researchers are never value free either! We have our own goals, histories and values, and these will inevitably affect what and how we prefer to research.

International sharing, networking and awareness need encouraging

At an international conference such as this, and despite the fact that many people here are working on the same lines, there will inevitably be similarities and differences between us. This should not be considered as a problem but welcomed. We all develop our ideas by experiencing contrasts, and thus we should be celebrating and valuing diversity and enjoying the challenging contrasts such a conference provides. In the same way we should all of us beware of cultural/linguistic imposition. Regrettably I am guilty of imposing my language on you all, and I therefore finish by apologising for that. Nevertheless I hope that you will forgive me, and also that you try to see through the barriers of languages to consider the ideas which I have presented to you.

I hope you all have an enjoyable and stimulating conference.

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PROFESSIONAL DEVELOPMENT: AN AUSTRALIAN CASE: THE POWER OF ONE-TO-ONE ASSESSMENT INTERVIEWS

Doug Clarke
Australian Catholic University (Melbourne)
d.clarke@patrick.acu.edu.au

In this paper, I outline what I see as the benefits to teachers' professional development of the use of task-based, one-to-one assessment interviews with students of early and middle years mathematics. I draw upon data from the Victorian Early Numeracy Research Project, our recent work in the domain of rational numbers, and examples from interviews with students in USA and Australia. Such interviews enhance knowledge of individual and group understanding of mathematics, and assist teachers in lesson planning and classroom interactions as they gain a sense of typical learning paths. I argue that an appropriate prelude to lesson study is gaining data on what students know and can do in particular mathematical domains (individually and in a group sense). Large-scale collection of data of this kind also has potential to inform curriculum policy and guidelines.

Background

In the last twenty years, assessment in the early and middle years of schooling has been characterised by a shift in the balance between the summative and formative modes. The inadequacy of a single assessment method administered to students at the end of the teaching of a topic is widely acknowledged. It is increasingly the case that teachers and school administrators regard the major purpose of assessment as supporting learning and informing teaching.

Other reasons for an expansion in assessment methods include a broadening of those skills and understandings which are valued by teachers, schools and educational systems. For example, in the publication, Adding It Up (Kilpatrick, Swafford, & Findell, 2001), the term "mathematical proficiency" was introduced, which the authors saw as including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The limitations and disadvantages of pen and paper tests in gathering accurate data on children's knowledge were well established by Clements and Ellerton (1995). They contrasted the quality of information about students gained from written tests (both multiple-choice and short-answer) with that gained through one-to-one interviews, and observed that children may have a strong conceptual knowledge of a topic (revealed in a one-to-one interview) but be unable to demonstrate that during a written assessment.

The findings of this research contrast with the continued common emphasis in many classrooms today on procedural fluency. Reading issues in written tests are also of great significance.

For the past fifteen years, it has become common for teachers of literacy to devote time to assessing students individually, and using the knowledge gained to teach specific skills and strategies in reading (Clay, 1993; Hill & Crevola, 1999). The late 1990s, in Australia and New Zealand, saw the development and use of research-based one-to-one, task-based interviews on a large scale, as a professional tool for teachers of mathematics (Bobis, Clarke, Clarke, Gould, Thomas, Wright, & Young-Loveridge, 2005).

I outline below examples from two projects and the experiences of the authors in developing, piloting, and using interviews within professional development contexts. The potential of such interviews for enhancing teacher content knowledge and knowledge for teaching (Hill & Ball, 2004) is discussed. It will be argued that the use of such interviews can enhance many aspects of teacher knowledge, with consequent benefits to students.

The Early Numeracy Research Project

The Early Numeracy Research Project (ENRP) research and professional development program conducted in Victoria from 1999 to 2001 in Years Prep to 2 (with some limited data collection of the original Prep cohort in Years 3 and 4, in 2002 and 2003 respectively), investigated effective approaches to the teaching of mathematics in the first three years of schooling, and involved teachers and children in 35 project ("trial") schools and 35 control ("reference") schools (Clarke, 2001; Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). In all, the project involved 353 teachers and over 11 000 students of ages 4 to 8.

There were three key components to this research and professional development project:

- the development of a research-based framework of "growth points" in young children's mathematical learning (in Number, Measurement and Geometry);
- the development of a 40-minute, one-on-one interview, used by all teachers to assess aspects of the mathematical knowledge of all children at the beginning and end of the school year (February/March and November respectively); and
- extensive professional development at central, regional and school levels, for teachers, coordinators, and principals.

As part of the ENRP, it was decided to create a framework of key "growth points" in numeracy learning. Students' movement through these growth points in trial schools, as revealed in interview data, could then be compared to that of students in the reference schools. The project team studied available research on key "stages" or "levels" in young children's mathematics learning (e.g., Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; Mulligan & Mitchelmore, 1996; Owens & Gould, 1999; Wilson & Osborne, 1992; Wright, 1998; Young-Loveridge, 1997), as well as frameworks developed by other authors and groups to describe learning.

The decision was taken to focus upon the strands of Number (incorporating the domains of Counting, Place value, Addition and subtraction strategies, and Multiplication and division strategies), Measurement (incorporating the domains of Length, Mass and Time), and Geometry (incorporating the domains of Properties of shape, and Visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There were typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children "count-all" to find the total ("1, 2, 3, ..., 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and counting on ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting All and Counting On are therefore two important growth points in children's developing understanding of Addition.

These growth points informed the creation of interview tasks, and the recording, scoring and subsequent data analysis, although the process of development of interview and growth points was very much a cyclical one. In discussions with teachers, I have come to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide a kind of mapping of the conceptual landscape. However, I do not claim that all growth points are passed by every student along the way.

The one-to-one interview was used with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February/March and November respectively), over a 30- to 50-minute period, depending upon the interviewer's experience and the responses of the child. The interviews were conducted by the classroom teacher in trial schools, and a team of interviewers in reference schools. A range of procedures was developed to maximise consistency in the way in which the interview was administered across the 70 schools.

Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child moved through all of these. The interviewer made a decision after each task. Given success, the interviewer continued with the next task in the domain as far as the child could go with success. Given difficulty with the task, the interviewer either abandoned that section of the interview and moved on to the next domain or moved into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

The interview provided information about growth points achieved by a child in each of the nine domains. Below are two questions from the interview. These questions focus on identifying the mental strategies for subtraction that the child draws upon. The strategies used were recorded on the interview record sheet.

> 19) Counting Back
> For this question you need to listen to a story.
> a) Imagine you have 8 little biscuits in your play lunch and you eat 3.
> How many do you have left? ... How did you work that out?
> *If incorrect answer, ask part (b):*
>
> b) Could you use your fingers to help you to work it out? *(It's fine to repeat the question, but no further prompts please).*
>
> 20) Counting Down To / Counting Up From
> I have 12 strawberries and I eat 9. How many are left? ... Please explain.

It was intended that the interview would provide a challenge for all children. Over 36,000 interviews were conducted by teachers and the research team during the ENRP, and only one child was successful on every task — a Grade 2 boy in the second year of the project. It appeared that the aim of challenging all was achieved, with one possible exception!

Australian Catholic University Rational Number Interview

Following the perceived success of the Early Numeracy Research Project, it was decided to develop a one-to-one interview for teachers of nine- to fourteen-year olds. Given the recognised difficulty with fractions and decimals for many teachers and students (see, e.g., Behr, Lesh, Post, & Silver, 1983; Kieren, 1988; Lamon, 1999; Steinle & Stacey, 2003), it was decided to make rational numbers the focus of the interview. Anne Roche adapted and developed tasks in decimals (see, e.g., Roche, 2005; Roche & Clarke, 2004) and Annie Mitchell in fractions (see Mitchell & Clarke, 2004; Mitchell, 2005). In 2005, Clarke, Roche and Mitchell collaborated with Jan Stone (Association of Independent Schools, New South Wales) and Professor Richard Evans (Plymouth State University) in refining these tasks. A major source of tasks included the Rational Number Project (Behr & Post, 1992).

Once again, the selection of tasks used by the teacher is made during the interview, according to students' responses. There are currently 31 tasks assessing fraction understanding, 14 assessing decimal understanding, and 3 assessing proportional reasoning. Development on a range of tasks for percentages is continuing. To this point, approximately 70 teachers have been involved in piloting the tasks with their students. Two sample tasks are given in Figure 1.

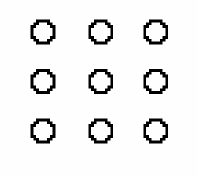
Nine dots	**Ordering**
Show the student the picture of 9 dots.  If this is three-quarters of a set of dots, how many dots is two-thirds of the set? *(drawing is okay if necessary)* ………. Please explain your thinking. [adapted from Cramer & Lesh, 1988]	*Place the cards randomly on the table.* Put these numbers in order from smallest to largest. *Encourage the student to think out loud while ordering them* **a) 0 0.01 0.10 .356 0.9 1 1.2** *Show each card below in turn* **1.70 1.05 .10** b) Where would this decimal go? Why does it belong there?

Figure 1. Sample tasks from the Australian Catholic University Rational Number Interview.

It should be noted that the task of developing "growth points" or a learning and assessment framework in rational number understanding is proving more elusive than for the domains of the ENRP. At present, our compromise is a statement of 25 "big ideas" in rational number knowledge, skills and understanding.

For example, one big idea is "works within a variety of physical and mental models (areas and regions, sets, number lines, ratio tables, etc.), in continuous and discrete situations." However, because the domain of rational numbers is made up of many aspects or "subconstructs" (Kieren, 1988), and the use of many models within each subconstruct (Lamon, 1999), it has been a challenging task to try to map out a "conceptual landscape" for this content.

Similarly, it has been difficult to arrange the interview tasks in the same way as the ENRP, with many "drop-out points" and detours, as I have found that success or lack of success on a given task is not necessarily a good predictor for performance on another task, even when they seem closely related.

In the following sections, particular tasks and insights from teachers will be used to build the argument of the power of the interview as a professional development tool. I will outline the benefits to teacher professional growth and therefore the quality of teaching of the use of task-based, one-to-one interviews by mathematics teachers in the early and middle years of schooling.

INTERVIEWS AS A POWERFUL TOOL FOR MATHEMATICS TEACHERS

In the remainder of the paper, I will use data collected from teacher surveys as supporting data, and anecdotes from our own experience, a combined total of approximately 500 interviews.

Higher quality assessment information

In contrast to the traditional pen and paper test, a carefully-constructed and piloted one-to-one interview can provide greater insights into what students know and can do. Student strategies are recorded in detail on the interview record sheet. For example, in addition and subtraction, for the two subtraction tasks outlined earlier in this article, the teacher completes the record sheet, as shown in Figure 2, recording both the answer given and the strategies used. The emphasis on recording both answer and strategies is clear recognition that the answer alone is not sufficient.

The act of completing the record sheet requires an understanding of the strategies listed (e.g., modelling all, fact family, count up from, etc.). The use of the interview is therefore building pedagogical content knowledge (Shulman, 1987).

The capacity of the teacher to take the information on the record sheet and "map" student performance in relation to the growth points or "big ideas" is a key step in the process. Teachers after conducting the interview are likely to ask the reasonable question in relation to planning, "So now what?" If they have a clear picture of individual and group performance in particular mathematical domains, they are then in a position, hopefully with support of colleagues, to plan appropriate classroom experiences for individuals and groups.

19. Count Back / Modelling All (8 – 3)

a. Answer .... ☐

- ***Known fact or fact family (eg., 5 + 3 = 8)***
- ***Count back all, in head (7, 6, 5 or 8, 7, 6, 5)***
- ***Count back all, with fingers used to keep track only (7, 6, 5 or 8, 7, 6, 5)***
- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

b. Answer .... ☐

- Modelling all (shows 8 fingers, then takes away 3)
- Other ....

20. Count Down To / Count Up From (12 – 9)

Answer .... ☐

- ***Known fact or fact family (eg., 9 + 3 = 12)***
- ***Count down to (12, 11, 10, 9)***
- ***Count up from (9, 10, 11, 12)***
- ***Fingers used during "count down to" or "count up from" to keep track only***
- Count back all (12, 11, 10, 9, 8, 7, 6, 5, 4, 3)
- Modelling all (shows 12 "things", then takes away 9 "things", leaving 3)
- Other ....

Figure 2. An excerpt from the addition and subtraction interview record sheet.

A focus on mental computation

Northcote and McIntosh (1999), using surveys of all reported computations of 200 adults over a 24-hour period, concluded that approximately 83% of all computations involved mental methods, with only 11% involving written methods. In addition, they found that over 60% of all computations only involved an estimate. These findings influenced greatly the construction of our interviews, where mental computation and estimation feature prominently.

Physical involvement: Making the task match the desired skill

Some mathematical skills and understandings can be very difficult to assess without some kind of physical task. As one teacher wrote, "to see whether children can do physical things, we sometimes need to watch." Consider this task from the Place Value section of the ENRP interview. The child is given a pile of icy-pole sticks, 7 bundles of 10 sticks each wrapped in an elastic band and about 20 loose ones. The teacher explains to the child that there are "bundles of ten and some more loose ones." The child is then shown a card with the number "36" on it, and asked to "get this many icy-pole sticks."

The student response is a very helpful indicator of place value understanding, in that some will feel the need to pull apart the bundles of ten (possibly indicative of an understanding of 36 only as the number in the sequence 1, 2, …, 36); some will count “10, 20, 30,” and then “1, 2, 3, 4, 5, 6.” A subtle improvement is the child who is able to say “3 of these and 6 of those,” without any need to count. It is difficult to imagine a task that didn’t involve this level of physical action providing the same opportunity for the teacher to gain what they do from listening and watching.

Objects of various kinds also increase the level of accessibility to tasks, and enjoyment of the experience for the student. There is also a number of topics in the mathematics curriculum which are not easily assessed by traditional means, e.g., visualisation and orientation, and manipulation of objects allowed students to show what they know.

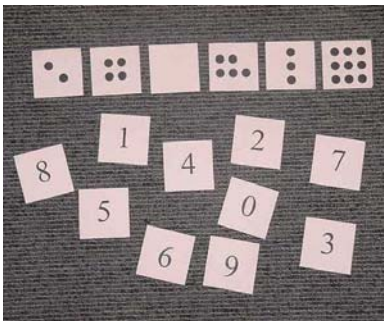
Large scale valid and reliable data

Processes used by the research team to maximise reliability and validity of interview data have been detailed elsewhere (see Clarke, 2001, Clarke et al., 2002). Having data on over 36 000 ENRP interviews across Grades Prep to 4 (the project focused on Grades Prep to 2, but a small “spin-off” project involved interviews with over 1000 students at each of Grades 3 and 4), provided previously-unavailable high quality data on student performance. These data had several benefits:

- Information for teachers on what “typical performance” for various grade levels looked like enabled them to relate the performance of their students to that of the cohort. For example, Table 1 shows the percentage of children on arrival at schools in trial schools who were able to match numerals to their corresponding number of dots. The data on children in the first year of school is discussed in considerable detail in Clarke, Clarke and Cheeseman (2006).

  Table 1. Performance of trial school children on entry to school in February 2001 on selected tasks (%) (n = 1437)

	Percent Success
Match numeral to 2 dots	86%
Match numeral to 4 dots	77%
Match numeral to 0 dots	63%
Match numeral to 5 dots	67%
Match numeral to 3 dots	79%
Match numeral to 9 dots	41%

  Teachers and researchers found considerable variation within classes in what students knew and could do, to an even greater extent than many previously thought. Of course, this makes a mockery of arguments that “all Prep children should be studying this and not that.”

- Information is available for state departments of education and curriculum developers to inform their work. One of the most powerful pieces of data which is hopefully informing the development of the Victorian Essential Learning Standards (Victorian Curriculum and Assessment Authority, 2005) is found in the domain of Addition and Subtraction. Achievement of the growth point “Derived strategies in addition and subtraction” was assessed by the following tasks:

12 - 6   7 + 8   19 - 15   16 + 5   36 + 9

- Students were deemed to have achieved the growth point if they answered correctly (mentally, with no time limit), and used at least three preferred strategies across the five problems. For example, for 36 + 9, counting by ones (“36, 37, 38, …, 45”) is a non-preferred strategy, while 36 + 10 - 1 would be a preferred strategy.

  At the end of Grade 2, only 19% of “typical children” could succeed on this basis. Even in trial schools (where teachers had been given intensive professional development), the percentage was only 31%. Yet, at the time, the state curriculum guidelines implied that virtually all children should be able to do these tasks. In light of these data (and the figure for typical students at the end of Grade 4—55%), it would appear that the state curriculum needs revision in terms of this content, as well as a consideration of whether the common practice of introducing conventional algorithms as early as Grade 2 is completely inappropriate (see Clarke, 2005 for more on this issue).

Building a knowledge of variations in performance across grade levels

It is interesting to collect sufficient data in order to observe trends in development of student understanding across the grade levels. To illustrate this point, a task adapted from the Rational Number Project (Cramer, Behr, Post & Lesh, 1997; Cramer & Lesh, 1988) and used in our Rational Number Interview, is shown in Figure 3.

Figure 3. A task used in the Australian Catholic University Rational Number Interview.

Fraction pie

Show the student the pie diagram.

a) What fraction of the circle is part B?.....
How do you know that?
b) What fraction of the circle is part D?.....
How do you know that?

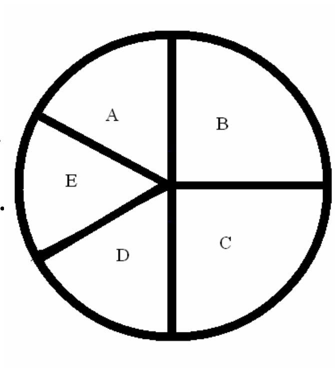


Table 2 shows student performance by grade level on the two parts of this task. To be correct, both the correct answer and an appropriate explanation were required. Students who were unsuccessful on part (a) were not given part (b) to attempt. Once again, the difficulty posed by this task for many students, possibly due to a lack of familiarity with tasks where not all parts are the same size, has implications for both emphasis and the pace of moving through content in fractions.

Table 2. Student Performance on Part-Whole Task (Continuous Case) by Grade Level (Years 4-6)

Q4 Part B Pie			
Grade	4	5	6
Correct	35/58	52/68	50/61
%	60%	76%	82%

Q4 Part D Pie			
Grade	4	5	6
Correct	29/58	36/68	33/61
%	50%	53%	54%

One of the advantages of administering the assessment interview at both the beginning and end of the school year was that teachers were provided, face-to-face, with exciting evidence of growth in student understanding over time.

Relating performance in one part of the interview to performance in another part

It is informative for teachers and researchers to consider whether understandings evident in one part of the interview prove accessible in another context. A major feature of teaching for relational understanding (Skemp, 1976) is that understanding enhances transfer (Hiebert & Carpenter, 1992). Among the possible tasks a student might encounter in the ENRP interview Counting section were tasks asking them to count by 2s, 5s, and 10s from 0. Given success, they counted by 10s and 5s, from 23 and 24 respectively.

In the Multiplication and division part of the interview, as part of a task assessing what we called abstracting multiplication and division (see Sullivan, Clarke, Cheeseman, & Mulligan, 2001), students were shown an array of dots which was then partially-covered as shown in Figure 4. They were then asked: “How many dots altogether on the card?” Even when students who counted by ones were prompted by, “could you do it a different way, without counting them by ones,” the success rate was not high. Only 37.5% of 2942 Year Prep to 2 students were successful in transferring those skills to this new context.

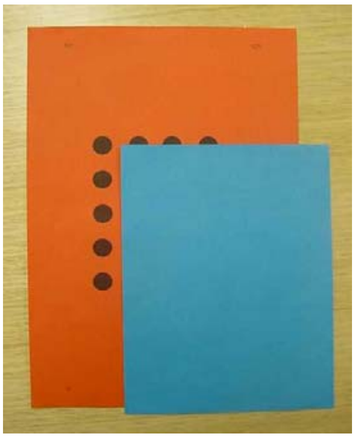


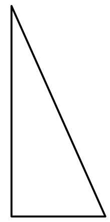
Figure 4. An array task.

Enhanced teacher knowledge of mathematics

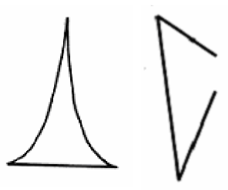
Our experience in working with teachers is that the use of the interviews enhances teacher content knowledge. In the middle years, many teachers acknowledge their lack of a connected understanding of rational number, often using limited subconstructs (sometimes only part-whole), and limited models (such as the ubiquitous “pie”). Many teachers have reported that their own understanding of rational number (e.g., an awareness of subconstructs of rational number such as measure and division and the distinction between discrete and continuous models) has been enhanced as they observe the variety of strategies their students draw upon in working on the various tasks and complete the record sheet.

Some might presume that teacher content knowledge is not an issue. However, many teachers reported that terms such as “counting on,” “near doubles”, and “dynamic imagery” were unfamiliar to them, prior to their involvement in the ENRP. It is interesting to consider whether this is content knowledge or “knowledge for teaching” (see, e.g., Ball & Bass, 2000; Ball & Hill, 2002; Hill & Ball, 2004).

Teachers develop an awareness of the common misconceptions and strategies which they may not currently possess



As teachers have the opportunity to observe and listen to students’ responses, they become aware of common difficulties and misconceptions. For example, many children in Years Prep to 4 were unable to give a name to the shape on the left. It wasn’t expected that they would name it “right-angled triangle,” but simply “triangle”. Because it didn’t correspond to many students’ “prototypical view” (Lehrer & Chazan, 1998) of what a triangle was (*a triangle has a horizontal base and “looks like the roof of a house”—either an isosceles or equilateral triangle*), some called it a “half-triangle, because if you put two of them together you get a real triangle.” Many students nominated the two shapes on the right as triangles.



It was clear from a teaching perspective that it was important to focus on the properties of shapes, and to present students with both examples and non-examples of shapes.

The quiet achievers sometimes emerge

> *In every class there is that quiet child you feel that you never really ‘know’—the one that some days you’re never really sure that you have spoken to. To interact one-to-one and really ‘talk’ to them showed great insight into what kind of child they are and how they think (ENRP teacher, March 1999, quoted in Clarke, 2001).*

In response to a written question on highlights and surprises from the Early Numeracy Interview, a number of teachers noted that the one-to-one interview enabled some “quiet achievers” to emerge, and several noted that many were girls. There appeared to be some children who didn’t involve themselves publicly in debate and discussion during whole-class or small-group work, but given the time one-to-one with an interested adult, really showed what they knew and could do.

> *The greatest highlight was that no matter at what level the children were operating mathematically, all children displayed a huge amount of confidence in what they were doing. They absolutely relished the individual time they had with you; the personal feel, and the chance to have you to themselves. They loved to show what they can do (ENRP teacher, March 1999, quoted in Clarke, 2001).*

Improved teacher questioning techniques (including the use of wait time)

Teachers noted that the interview provided a model for classroom questioning, and as a result of extensive use of the interview, they found themselves making increasing use of questions of the following kind:

- Is there a quicker way to do that?
- How are these two problems the same and how are they different?

- Would that method always work? . . .
- Is there a pattern in your results?

Teachers also observed the power of *waiting* for children's responses during the interview, noting on many occasions the way in which children who initially appeared to have no idea of a solution or strategy, thought long and hard and then provided a very rich response. Such insights then transferred to classroom situations, with teachers claiming that they were working on allowing greater wait time.

Tasks provide a model for classroom activities

Teachers were strongly discouraged from "teaching to the test" through presenting identical tasks to those in the interview during class. Nevertheless, the tasks did provide a model for the development of different but related classroom activities. For example, in the Place Value section of the Early Numeracy Interview, students are asked to type numbers on the calculator as they are read by the teacher or read numbers that emerge as they randomly pick digits and extend the number of places (ones, tens, hundreds, etc.) of the number on the screen.

Seeing the potential of the calculator as a tool for exploring and extending place value understanding, teachers would try tasks such as "type the largest number on the calculator which you can read (but no zeros in it)." The reason for the instruction to have no zeros in the number was because some children will be able to read a million, but not necessarily 386. Such a task provides an opportunity for the teacher to challenge them to make the number even larger. This task, re-visited regularly, provides a helpful measure of growth in student understanding over time, and therefore can be used as an ongoing assessment tool.

Teacher professional growth: Some final comments

At a professional development day involving all 250 or so teachers) towards the end of 1999, ENRP teachers were asked to identify changes in their teaching practice (if any), as a result of their involvement in the project. There were several common themes, many of which can be related to the professional growth experienced through the use of the interview:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- provision of more time to explore concepts;
- greater opportunities for children to share strategies used in solving problems;
- provision of greater challenges to children, as a consequence of higher expectations;
- greater emphasis on "pulling it together" at the end of a lesson, as part of a whole-small-whole approach;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and "real life mathematics".
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

Several of the themes discussed in this article are evident in the following quote from a teacher who attended the professional development program:

> *The assessment interview has given focus to my teaching. Constantly at the back of my mind I have the growth points there and I have a clear idea of where I'm heading and can match activities to the needs of the children. But I also try to make it challenging enough to make them stretch.*

ONE –TO-ONE INTERVIEWS AND LESSON STUDY

So what is the potential relationship between the use of one-to-one assessment interviews and lesson study? In describing the Early Numeracy Research Project, we have sometimes used these words: "understanding, assessing and developing young children's mathematical thinking."

The growth points provide a way of *understanding* students' thinking and possible pathways or trajectories through which students might move, the interview provides a way of establishing where students "are at" in relation to these pathways (*assessing*), and the professional development program provided an opportunity to explore how this understanding might be developed further ("*developing*"). I would argue that lesson study fits very nicely in with the third aspect. If teachers have a clear picture of their students' understanding of mathematics and a framework against which this can be mapped, then lesson study provides an ideal model for planning "where to from here?" In this way, the use of one-to-one assessment interviews is in complete harmony with the lesson study approach.

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IMPLEMENTING LESSON STUDY
IN NORTH AMERICAN SCHOOLS AND SCHOOL DISTRICTS

Akihiko Takahashi

DePaul University

Because no professional development practice similar to lesson study exists in North America, it is often challenging for North American teachers and schools to implement lesson study. Lesson study has, however, become highly visible in many state, national, and international conferences, open houses, high-profile policy reports, and special journal issues in North America. Moreover, numerous schools and school districts in the United States have attempted to use it to change their practices and to impact student learning. This paper is intended to provide some ideas about how to conduct lesson study for the educators who are interested in implementing lesson study in their schools and school districts.

JAPANESE LESSON STUDY MODEL

The practice of lesson study originated in Japan. Widely viewed in Japan as the foremost professional development program for teachers, lesson study is credited with dramatic success in improving classroom practices in the Japanese elementary school system (Fernandez, Chokshi, Cannon, & Yoshida, 2001; Lewis, 2000; Lewis & Tsuchida, 1998; Shimahara, 1999; Stigler & Hiebert, 1999; A. Takahashi, 2000; Yoshida, 1999).

A particularly noticeable accomplishment in the past 20 years of lesson study in Japan has been the transformation from teacher-directed instruction to student-centered instruction in mathematics and science (Lewis & Tsuchida, 1998; Takahashi, 2000; Yoshida, 1999). The success of lesson study can be found in two primary aspects: improvements in teacher practice and the promotion of collaboration among teachers.

First, lesson study embodies many features that researchers have noted are effective in changing teacher practice, such as using concrete practical materials to focus on meaningful problems, taking explicit account of the contexts of teaching and the experiences of teachers, and providing on-site teacher support within a collegial network. It also avoids many features noted as shortcomings of typical professional development, e.g., that it is short-term, fragmented, and externally administered (Firestone, 1996; Huberman & Guskey, 1994; Little, 1993; Miller & Lord, 1994; Pennel & Firestone, 1996). In other words, lesson study provides Japanese teachers with opportunities to make sense of educational ideas within their practice, to change their perspectives about teaching and learning, and to learn to see their practice from children’s perspectives. For example, a Japanese teacher said, “It is hard to incorporate new instructional ideas and materials in classrooms unless we see how they actually look. In lesson study, we see what goes on in the lesson more objectively, and that helps us understand the important ideas without being overly concerned about other issues in our own classrooms" (Murata & Takahashi, 2002).

Second, lesson study promotes and maintains collaborative work among teachers while giving them systematic intervention and support. During lesson study, teachers collaborate to: 1) formulate long-term goals for student learning and development; 2) plan and conduct lessons based on research and observation in order to apply these long-terms goals to actual classroom practices for particular academic contents; 3) carefully observe the level of students' learning, their engagement, and their behaviors during the lesson; and 4) hold post-lesson discussions with their collaborative groups to discuss and revise the lesson accordingly (Lewis, 2002). One of the key components in these collaborative efforts is "the research lesson," in which, typically, a group of instructors prepares a single lesson, which is then observed in the classroom by the lesson study group and other practitioners, and afterwards analyzed during the group's post-lesson discussion. Through the research lesson, teachers become more observant and attentive to the process by which lessons unfold in their class, and they gather data from the actual teaching based on the lesson plan that the lesson study group has prepared. The research lesson is followed by further collaboration in the post-lesson discussion, in which teachers review the data together in order to: 1) make sense of educational ideas within their practice; 2) challenge their individual and shared perspectives about teaching and learning; 3) learn to see their practice from the student's perspective; and 4) enjoy collaborative support among colleagues (Akihiko Takahashi & Yoshida, 2004).

Lynn Liptak, a principal who is pioneering lesson study in the U.S., argues that because lesson study is a teacher-led approach to professional development, teachers can be actively involved in the process of instructional change, in contrast with traditional professional development methods.

Contrasting methods of professional development

Traditional	Lesson Study
Begins with answer	Begins with question
Driven by outside "expert"	Driven by participants
Communication flow: trainer to teachers	Communication flow: among teachers
Hierarchical relations between trainer & learners	Reciprocal relations among learners
Research informs practice	Practice is research

(Reprinted from Lewis, 2002, p.12)

Lesson study also has played an important role in improving curricula, textbooks, and teaching and learning materials in Japan. In fact, most Japanese mathematics textbook publishers employ as authors classroom teachers who are deeply involved in lesson study, and their materials are in some manner examined through the process of lesson study.

The process of lesson study

Lesson study does not follow a uniform system in Japan. It is more like a cultural activity. As a result, lesson study takes many different forms, including school-based lesson study, district-wide lesson study, and cross-district lesson study. Therefore, there are neither clear definitions nor specified criteria of lesson study in Japan. Its process differs across schools, districts and types of lesson study. Lesson study groups can be formed by all the members in a school building, or by study-group members in a district, or by teachers who are interested in specific subject matter.

Although the types of groups differ, the lesson study process usually begins with identifying a long-term goal or goals or a research question or set of questions as a theme. Since lesson study is a way to bring educational goals and standards to life in the classroom (Lewis, 2002), this process usually involves all the members of the lesson study group. After a lesson study group establishes a theme, the cycles lesson study begin. A typical lesson study group activity involves several lesson study cycles in a year. A lesson study group usually divides into two or more sub groups each containing four to six teachers sharing a particular interest or teaching the same or similar grade levels. One of the sub groups, called the "lesson planning team," develops a lesson plan and conducts a research lesson. The other sub-group members who are not involved in planning the lesson but who observe the lesson, are called "research lesson participants." In each lesson study cycle, a different sub-group becomes the lesson planning team. A lesson study group sometimes invites teachers and university professors from outside the group as lesson study participants. Both lesson planning team members and lesson study participants play important roles and contribute differently to the lesson study project.

A major role of the lesson planning team is to develop a lesson plan. Based on this lesson plan, one of the teachers from the team teaches his or her class. This lesson is called the "research lesson" (*Kenkyuu-jugyou*) and is observed by all the members of the lesson study group. To develop a lesson plan, the group usually meets three to five times for sessions of two to three hours. The team members also prepare teaching and learning materials such as manipulatives and student worksheets for the lesson.

Following the research lesson, the lesson planning team and all the research lesson participants discuss whether the students in the class accomplish the goal or goals of the lesson. This is called post-lesson discussion (*Kenkyu-kyogikai*). A major role of the research lesson participants is to study the impact of the lesson in order to improve the lesson plan. To do this they need to collect data during the research lesson to support their arguments. Participants might collect various types of data, such as how many students actually solved the problem and how many different solution methods were discussed in the class, how particular students solved the problem during the lesson and how the class discussion helped these students to improve their solution methods, or how particular students summarized the class discussion in their notes. Participants may collect data differently depending on their interests and experiences. They may also interpret the data differently. As a result, a wide variety of data can be expected for a post-lesson discussion and will contribute to the richness of the post-lesson discussion and greatly help to improve the lesson plan. In this way each research lesson participant is expected to be like a researcher who collects data to examine whether the lesson plan facilitates student learning and whether the lesson plan need to be improved. The lesson planning team also plays an important role during the post-lesson discussion. They are expected to explain the discussion and rationale behind the lesson plan. This information helps participants better understand the lesson.

The activities of lesson study -- reading a lesson plan, observing a class, and examining the class in terms of student learning -- all benefit the research lesson participants in their larger roles as classroom teachers when they develop their own lesson plans and work to improve their own instruction.

Outside specialists (*Koshi*), so-called knowledgeable others, may also play an important role in lesson study. The knowledgeable other is typically invited as an advisor for the lesson planning team and as an outside commentator who summarizes the post-lesson discussion. Some schools and school districts engage the same knowledgeable others to continuously support their lesson study over a number of years. A lesson study group usually invites a person who has experience in the process of lesson study, and both pedagogical and content expertise, such as an experienced teacher, a university professor, or a district specialist. A knowledgeable other is expected not only to summarize the participants' discussion about the research lesson and draw out its important implications (Watanabe, 2002) but also to bring new perspectives to the lesson study group.

Throughout the lesson study process, teachers have opportunities to clarify how to apply particular educational ideas in their practice, to refine their perspectives on teaching and learning, to view their practices from the students' perspective, and to enjoy the collaborative support of their colleagues.

LESSON STUDY IN NORTH AMERICA

Many U.S. educators have recently become interested in lesson study as a promising source of ideas for improving education (Stigler & Hiebert, 1999). Within the last several years lesson study has become highly visible in many state, national, and international conferences, open-houses, high-profile policy reports, and special journal issues in North America. Moreover, some school districts in the United States have attempted to use it to change their practices and to impact student learning (Council for Basic Education, 2000; Germain-McCarthy, 2001; Research for Better Schools Currents Newsletter, 2000; Stepanek, 2001; Weeks, 2001).

 As of September 2, 2003 at least 29 states, 140 lesson study clusters/groups, 245 schools, 80 school districts, and 1100 teachers across the United States were involved in lesson study. (Lesson Study Research Group). The following map, figure 1, shows some lesson study groups in North America.

Figure 1

Chicago Lesson Study Group

One of many lesson study groups in North America, the Chicago Lesson Study Group has become well known among lesson study researchers and practitioners as one of the few groups that conduct public research lessons.

To explore the possibilities for replicating the success of Japanese lesson study in a U.S. setting, the Chicago lesson study group was launched in November of 2002, with volunteer school administrators and classroom teachers who have had university student teachers in their classrooms as a part of their field experiences. About twenty members are active and another thirty follow the group's activities on an email list Although the most popular form of lesson study in Japan takes place within a single school as a school-based professional development program (Yoshida, 1999), the Chicago Lesson Study Group adopted a cross-school volunteer model for its lesson study group. The reasons for this adaptation are twofold. First of all, an effective model of lesson study is often one that is started as a grassroots movement of enthusiastic teachers rather than as a top-down formation (Lewis 2002; Takahashi & Yoshida, 2004; Yoshida, 1999). For this reason, starting a lesson study group as a cross-school volunteer group was thought to be appropriate. Furthermore, it is sometimes difficult to establish a school-based lesson-study group in the U.S. because many teachers do not have experience working with other teachers in the same school as a group to accomplish a shared goal. Secondly, in order to have a sufficient number of enthusiastic elementary and middle school teachers who are interested in lesson study focusing on mathematics, a cross-school model was found to be more appropriate in the U.S. setting.

The program of activities for a volunteer lesson study group usually consists of two components: (1) a series of study groups concerned with improving the teaching and learning of mathematics (the group usually meets after school regularly throughout the year), and (2) several public research lesson opportunities each year to examine the work of the study group by inviting a wide variety of individuals to participate in its sessions. Since its inception, this study group has met twice a month to discuss ways to implement the ideas of reform mathematics in order to improve the teaching and learning of mathematics.

In order to find a way to implement the ideas of reform mathematics, the Chicago lesson study group has conducted five lesson-study conferences with ten public research lessons in the past four years. In each conference, the group has invited teachers and educators from not only the Chicago area but also from other states to discuss how to implement student-centered classrooms in mathematics. About one hundred participants from various U.S. states and Canada have attended the conferences each year and discussed how to help students develop algebraic thinking skills through problem solving.

HOW CAN WE BEGIN LESSON STUDY?

Because no professional development program similar to lesson study exists in the North America, it is often challenging for North American teachers and schools to implement lesson study. In order for teachers and schools to overcome the hesitation to become a part of lesson study, the following suggestions are usually given to the North American teachers and schools who are interested in exploring the possibility for implementing lesson study.

Begin with an informal study group

Since lesson study is a form of teacher-led professional development, any teacher can begin lesson study by connecting with another teacher. This means that lesson study is a grassroots movement among teachers rather than a top-down formation. Forming informal study groups focused on improving mathematics teaching and learning can be a step toward developing a lesson study group. If you are not already part of such group, you might share what happened in your math class during a grade level meeting. You do not have to begin lesson study with all the teachers in a school building at once. Forming a comfortable collaborative group is the most desirable step toward developing a lesson study.

Experience lesson study

The idea of lesson study is simple: collaborating with fellow teachers to plan, observe and reflect on lessons. Developing a lesson study, however, is a more complex process (Lewis, 2002). Because lesson study is a cultural activity, an ideal way to learn about lesson study is to experience it as a research lesson participant. In so doing, you will learn such things as how a lesson plan for lesson study is different from a lesson plan that you are familiar with, why such a detailed lesson plan is needed, what type of data experienced lesson study participants collect, and what issues are discussed during a post-lesson discussion. The following websites are excellent for exploration of the lesson study topics:

Chicago Lesson Study Group (http://www.lessonstudygroup.net)

Global Education Resources (http://www.globaledresources.com)

Lesson Study Group at Mills College (http://www.lessonresearch.net)

National Council of Teachers of Mathematics (http://nctm.org)

Identifying your research theme

Since lesson study is teacher-led professional development, the participants determine the group's theme or topic. For example, the Chicago Lesson Study Group chose measurement as their theme because measurement was the worst area of mathematics for their students as reflected in standardized test scores, and because measurement was the most difficult topic for them to teach. This theme emerged from a discussion about which topics teachers found difficult to teach.

Investigate a variety of materials to develop a lesson plan for a research lesson

Even though a group has identified its theme, it is still too early to develop a lesson plan. Some groundwork is needed. For example, if a group decides to explore how to teach measurement of the area of a rectangle for fourth grade students, the group needs to know how this topic relates to the other topics in the same grade, what prior knowledge students should have, and how this topic will help students learn mathematics in their future classes. Moreover, teachers need to know what kind of materials various textbooks use to teach this topic to students, and what research suggests (if anything) about various methods for teaching the topic. This investigation, called '*Kyouzai-kenkyuu*' in Japanese, means studying. Kyouzai-kenkyuu typically investigates the following areas:

- a variety of teaching and learning materials, such as curricula, textbooks, worksheets, and manipulatives
- a variety of teaching methods
- the process of student learning including students' typical misunderstandings and mistakes
- research related to the topic

Japanese teachers often begin *Kyozai-kenkyu* by comparing various teacher's guides published by textbook companies. Thus, U.S. teachers start using the English translation of the Japanese mathematics textbook series (Global Education Resources[1], 2006) and teaching guides for the Japanese Course of Study as resources to conduct *Kyozai-kenkyu*.

[1] http://www.globaledresources.com

Developing a lesson plan

There are many different types of lesson plans in Japan. Although no single universal form is available, any lesson plan is expected to provide enough information for lesson study participants to learn why the lesson-planning group decided to use a certain problem for the students, why the group chose a particular manipulative for the class, and why the group used particular wording for the key questions. To explain these rationales, a typical lesson plan includes the title of the lesson, the goal of the lesson, the relationship of the lesson to the standards or curriculum, the "about the lesson", the expected learning process, and the evaluation. Use of a simplified lesson plan might be a good idea for a novice lesson study group. One of the most difficult sections for teachers to develop in a lesson plan is the section describing the rationale of the lesson. Experienced participants often read this section very carefully because they believe that it is the essence of Kyozai-kenkyu. If this section cannot tell participants enough information about the lesson, the group's Kyozai-kenkyu might not be deep enough. Usually, the lesson plan rationale includes discussion of the following:

a. Concepts or skills that the students need to learn in the lesson or unit according to the standards and/or curriculum
b. Concepts or skills that the students have already learned
c. The major focus (theme) of this lesson or unit by comparing (a) and (b) (the objective of this lesson should be clearly stated)
d. The way to help students accomplish the above objective as a hypothesis for the research lesson

Lesson study groups might be able to test their draft lessons plan prior to the research lesson in another member's class as a pilot lesson. By using the data collected during the pilot lesson, the group might revise the lesson plan in preparing the research lesson.

The following shows a typical schedule for developing a lesson plan by a lesson planning team.

- The first meeting (five weeks before)
  Identifying the team's research goal/theme
  Deciding on a topic to investigate
- The second meeting (four weeks before)
  Investigate a variety of resources and teaching materials to develop a lesson plan (*Kyozai Kenkyu*)
- The third meeting (three weeks before)
  Developing a research lesson and writing the lesson plan for the lesson
- The fourth meeting (second weeks before)
  Completing the first draft of the lesson plan
- <Option: Teaching a class based on the first draft>
- The Fifth Meeting (a weeks before)

Completing the final draft and prepare for the lesson

Conduct a research lesson and a post-lesson discussion

Respecting the natural atmosphere of the class is always a priority during a research lesson, so ideally a research lesson should be held in the instructor's regular classroom. However, if the regular classroom cannot hold enough participants, a research lesson might be taught in a larger classroom. Further, out of respect for maintaining the natural environment, neither members of the lesson planning group nor participants should give any advice or comments to the students, because the instructor is the only person who can teach the students.

A post-lesson discussion is usually held right after the research lesson. It might be a good idea to have a post-lesson discussion in the classroom where the research lesson was held because participants can see all the blackboard writing and materials that the students used during the class. Customarily the post-lesson discussion session begins with an instructor's short comment on his or her teaching. An explanation of the lesson plan by a member of the lesson-planning group follows. Next, data collected by the participants may be discussed, followed by a more general discussion, which is sometimes focused on topics identified in advance. Although any critique and comments should be welcomed, a facilitator often keeps the discussion focused on the issues of interest to the planning group, rather than having a "free-for-all." At the end of the session, an outside specialist (Koshi) is given an opportunity to make a final comment as a summary of the session. The post-lesson discussion session should be recorded by a note taker. More guidelines for lesson observations and post-lesson discussions are available in *Lesson Study: A Handbook of Teacher-Led Instructional Change* (Lewis, 2002) and *Currents*, spring/ summer 2002 (Research for Better Schools, 2002).

LET'S BEGIN LESSON STUDY

Research suggests that mathematics class should be shifted from traditional teacher-led instruction to student-centered instruction. As a result, many schools and teachers are working hard to change their classrooms. However, most professional development programs are still done in a traditional way. The lesson study approach permits teachers involved in professional development to become as active in their learning as they expect their students to be.

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NESTING FEATURES OF DEVELOPING TEACHERS' PERSPECTIVES: A LESSON STUDY PROJECT FOR PROSPECTIVE TEACHERS IN MATHEMATICS WITH HISTORY AND TECHNOLOGY

Masami Isoda
University of Tsukuba, Japan
Email : isoda@criced.tsukuba.ac.jp

For teacher education with technology, we should consider many questions but there are no answers without focusing on the parameters. Here we discuss about a case for developing a good teacher's perspective through Lesson Study in terms of Japanese meaning with technology. Firstly, we define desirable teachers' perspectives. Secondly, we focus on the function of technology and history for teacher education. Thirdly, we analyze the case for explaining the developing process of teachers' perspectives in it.

I. Development of Teacher's Perspectives 'Kodomo wo miru me' for Mathematics

In the APEC meeting 'Innovative Teaching Mathematics through Lesson Study' in January 2006 at Tokyo, Catherine Lewis (2006) talked about her experience on Lesson Study in her keynote lecture as follows: (In her Lesson Study project) A U.S. teacher said as follows: "Before the Lesson Study, we had talked about multiple intelligence, constructivism and so on, but never talked about each subject matters of teaching. In the Lesson Study project, we began to talk about subject matters, why we teach them, how we teach them and what students learn from the lesson". In Tokyo's session, majority of participants may feel that this episode is not just for U.S. but for all countries. In the in-service teacher training programs, mathematics educators used to teach the theory of mathematics education. A comment from a teacher implicates that we teach theory and policy of curriculum and failed to teach them with subject matters. Multiple intelligence theory made us notice desirable competency which is not developed by one subject. In curriculum, teachers are expected to develop it through their lesson through teaching contents.

Constructivism theory promoted our awareness of the importance of listening students' ideas because students construct their knowledge by themselves. In teaching context, teacher's listening is not passive action such as only hearing but positive action (Arcavi & Isoda, to appear). Good lessons based on constructivism expect student-centralized lesson and the roles of teachers to conduct students' activity for their learning. In this context, listening activities by teachers are aimed to think about and find the way how to develop students' ideas to sophisticate or elaborate with others in their classrooms.

Lesson Study is an authentic activity for enabling teachers to conduct their classrooms. It includes discussions of subject matters, why they teach, how they teach and what students can learn.

Catherine also noticed her experience as follows; One teacher said, "I developed the eyes (teacher's perspective) to look at students and subject matters "Kodomo wo miru me". Now, I am well aware of my responsibility for my lesson. In the lesson study with other teachers, I preferred the more challenging lesson such as with Open-ended problems. When I found that students can challenge such difficult problems, I recognized self-confidence in my lessons". Catherine mentioned that teachers developed the ability to listen to students' ideas such as 'Kodomo wo miru me' but it is not only hearing (See such as Catherine

Lewis 2002). Because they developed good teachers' perspectives, they can say the development of eyes for understanding students and want to challenge the lesson with Open-ended problems and feel self-confidence through conducting the lesson.

Based on Japanese ideas of Lesson Study, teacher's perspective 'Kodomo wo miru me' is explained as the following (Isoda, Stephens, Ohara and Miyakawa, to appear): In Lesson Study, teachers discuss about the subject matter before the lesson. Teachers share responses (including misunderstandings) from students in the past lessons, have a lot of expectations about students' ideas and prepare their questions to extract students' ideas and their reaction against students' ideas. At the same time, teachers also expect that students' ideas will be more than their expectations. If students' ideas are within expectations, it is easily understandable for the teacher. Even if not, it is also within their expectations because it is a good chance for them knowing unknown ideas from students.

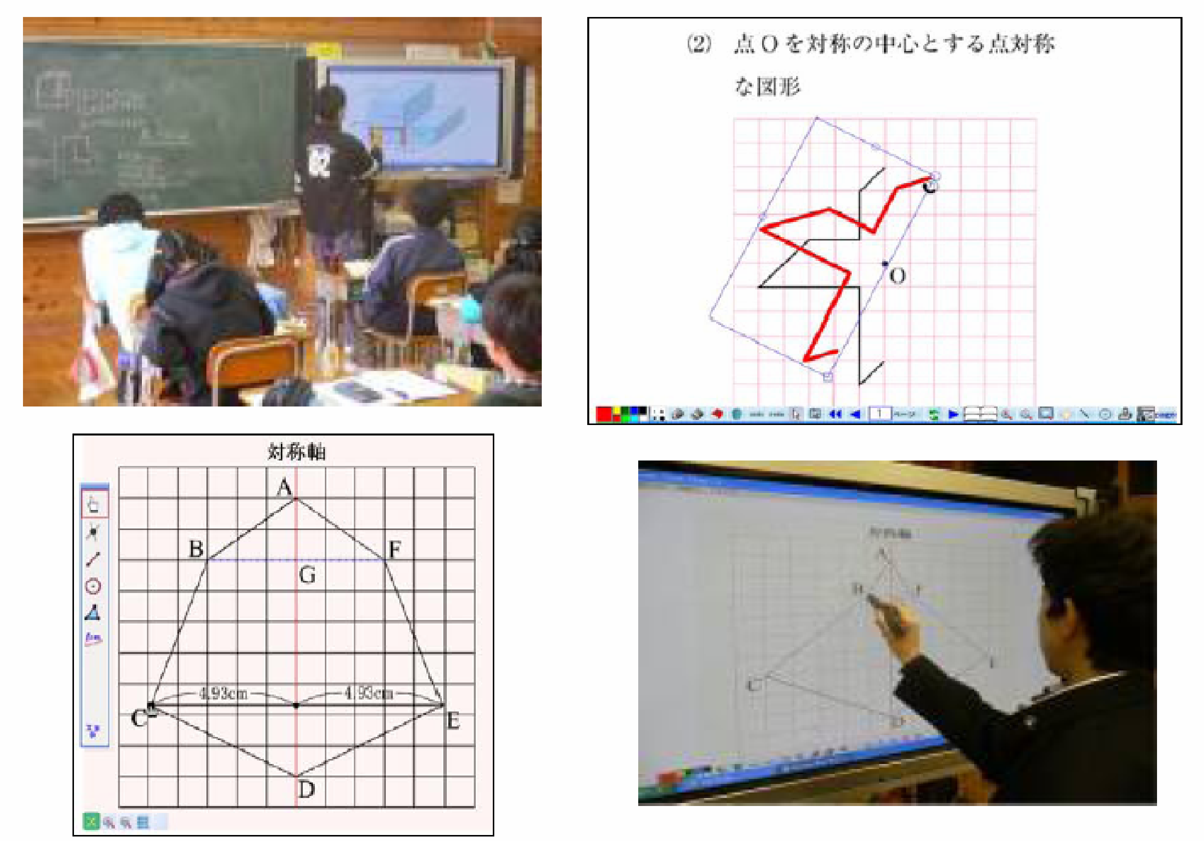
In teacher education, it is necessary to develop teachers for stepping up from listening to conducting. What necessary conditions for stepping up are and what kinds of processes are important for it even if there are no sufficient conditions. For example, some good teachers teach students the value what is important for life in any time and believe mathematics teaching is a part of the value education (Alan Bishop et al. 2003). Some novice teachers act differently between mathematics class and homeroom class. They worry how to solve and how to teach mathematics problems in every lesson but in homeroom activity, they try to push students' decision making. Through the experience, we can expect novice teachers to develop themselves to integrate their teaching contents and value. In this paper, the conditions and the processes are discussed as a case study.

II. Technology and History for Knowing Mathematics Differently

1. Minimum necessity to use technology for teacher education.

e-Larning is a current technology movement in education. Developing knowledge bank with learning management system is a trend. Equipment in schools and environment of internet are well known obstacles in general. But even if equipped, each teacher's belief of mathematics is an obstacle because mathematics is already embedded in physical or psychological tools such as papers, pencils and calculations. It is not easy to change teachers' beliefs because if we change tools then we have to change our mathematics itself. If we think their believes as an obstacle, we can not change. On contrary, If we recognize that each technological tool has it's own way of knowing mathematics differently, technology supports teacher educators to teach school mathematics differently and it may be a cue for next step.

For example, in mid of 90's, I engaged in in-service teacher training summer course to use a Graphing Calculator, Computer Algebra System and Dynamic Geometry Software during 5 years. The number of participants is more than fifty every year and half of them are repeaters. They enjoyed mathematics with technology, got the skill how to use and develop lesson plans for their classroom. But most of them did not use computers and graphing calculators in their classrooms because mathematics had been taught without technology and most subject matters in textbooks are not necessary to use technology. Between lines in textbooks, there are many things that should be taught. In a simple algebraic calculation from a line to a line, there are things which should be explained. Teacher can not alternate it to technology. In 90's, most of the technology developed as the environment and some mathematics educators believed to alternate the hidden aims in textbooks with technological environment. Indeed, we are now in a process of alternating textbooks to e-textbooks. The difference is that e-textbooks are a kind of textbooks. Teachers do not need to learn the commands how to use and integrate their aim of teaching with technological environment in the classroom (See Picture 1, Isoda et al. 2005).

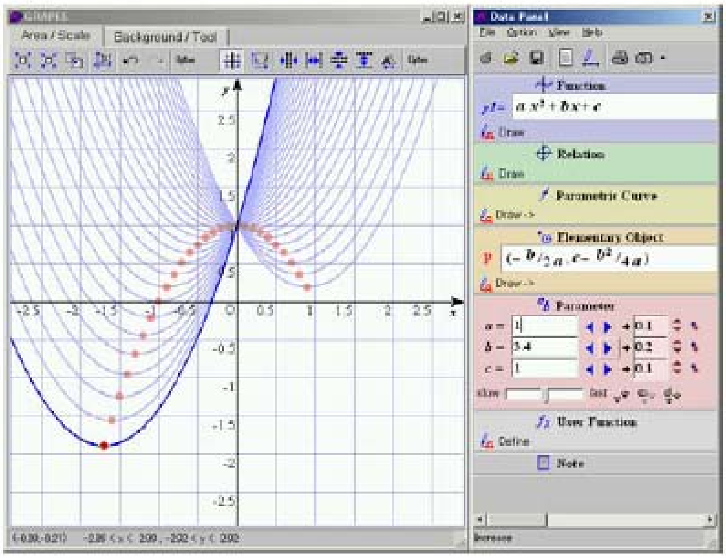


Picture 1. Using e-textbook with Interactive Board in classroom (Isoda et al. 2005)

What is obscure for me is that why many teachers had participated in summer courses even if they did not have a wish to alternate. I could say that they enjoyed knowing mathematics from different ways with technology. They enjoyed explorations of mathematics via technology. For example, if we draw graphs of $y = ax^2 + bx + c$ by fixing two parameters from a, b and c and changing one remained parameter regularly (Picture 2), we can find the role of each parameter, a, b or c which is never known by algebraic deduction to $y = (x-\alpha)^2 + \beta$.

Even if teachers did not have a chance to use computers or graphing calculators in their schools, exploring mathematics with technology in summer course is an enjoyable experience for them because it is the chance to know their known mathematics differently. If we use unknown technology, teachers can explore their school mathematics as unknown.

If we say that minimum necessity is needed to use technology in teacher education, we can say that it gives prospective or in-service teachers to explore school mathematics as a really new one. Teachers can re-experience their mathematics like students who learn from the beginning. Even if it is impossible because they already know, knowing differently is meaningful. If teachers know how to enjoy mathematics, it supports teachers enabling students to enjoy mathematics.



Picture 2. Grapes (Isoda et al. 2005)

2. Any Technology is innovative for knowing mathematics differently.

When we think about a function of technology in mathematics teacher education knowing mathematics differently, it is not necessary to focus on innovative technology because if we change technological or psychological tools (James Wertsch. 1991) we know mathematics differently. For example, if I have a card written with the number 2 in my left hand and I have a card with the number 6 in my right hand, and ask pupils to read cards, they must read two and six. If we bring closer both cards and ask the same question, what will happen? Pupils may begin to read twenty six. Even if we know that is the definition, we re-aware the difficulty and marvelous features of base ten system. Number Cards enable us to re-aware mathematics.

In elementary school mathematics, we usually use concrete materials for understanding. It is supported by not only Piaget' constructivism but also the theory of embodiment (George Lakoff, Raffael Nunez., 2000). For prospective teachers training, concrete materials are usually reused for teaching the methods of teaching because prospective teachers forgot how they learned content but prospective teachers enjoy like students before knowing it as the methods. For example, in picture 3 (MEXT, 2002), please find the price of an apple and the price of an orange posed with the picture without simultaneous equations. If you can solve it by operation of apples and oranges, you can enjoy unexpected explanation of the algebraic solution of simultaneous equations. Prospective teachers can recognize algebra as generalized operations of concrete objects.

Picture 3. How much each?

What implicates from these three examples here is that mathematical awareness is given with tools. Any technology for mathematics can be innovative for knowing mathematics differently.

3. Mathematics history as tools for cultural awareness

For knowing mathematics differently, mathematics itself can be useful. Indeed, mathematics is a psychological tool as for mediational means from the view point of Vygotskian theory (James Wertsch. 1991). History of mathematics itself is another mathematics when comparing with the current school mathematics. For mathematics teachers, I have been developing a web site in mathematics and history (See Picture 4. Isoda). It is not the web site of history itself. It's aim is to know mathematics differently and the origins from history. Most of contents are inspired

Picture 4. Mathematics History Museum by the Lesson Study Project (Isoda2005)

from historical texts in mathematics but with added educational view points. For example, in picture 4, it explains how to use sextant which was used for navigation before the age of radar and GPS. It tells us how high school mathematics was useful and necessary. A case study described in the next chapter is the Lesson Study Project that developed this website.

III. A Case Study of Developing Teachers' Perspective 'Kodomo wo miru me'

1. The introduction of Lesson Study in Japanese teacher education

It is difficult for prospective teachers to think like experienced teachers even if they take classes on a particular academic subject or on materials study. Thus, in teacher education programs in Japan, prospective teachers engage in micro-teaching exercises in which they engage in role playing, alternately playing the role of the teacher and the student to acquire the perspectives of both teacher and learner. They also participate in teaching internships of one month during which they do on-site training in an actual school. This allows students to become familiar with the cyclical Lesson Study process of researching materials, conducting Study Lessons, and holding feedback meetings to facilitate improvement. In the final week of their teaching internships, prospective teachers invite their advisors from the university to participate in their own Lesson Study project at the school.

2. A case study of Master Program in Education, University of Tsukuba

Becoming teachers by obtaining their Rank 1 Teaching Certificate in a master's degree program are trends in Japan. Each university's master's degree program offers its own excellent and distinctive teacher's education programs. Teacher education programs that cultivate the ability to lead practical and useful educational research are especially welcomed by teachers, the board of education, and the Ministry of Education, Culture, Sports, Science and Technology.

The Mathematics Course of the University of Tsukuba Master's Program in Education, which aims to train teachers for high school and beyond, addresses both pure mathematics and mathematics education. In the two year master program in education, we intend to develop leading teachers in mathematics education in school or university based on the tradition of ecole normale from 1873. Based on the image of leading teachers, following conditions are expected in this case study: 1) Good teachers can lead Lesson Study in their school, 2) Good teachers can teach other teachers how to use technology in mathematics from the beginning of his work, and 3) Good teachers can lead in the society of mathematics education.

In their first year of two year program, graduate students (prospective teachers) develop original mathematics teaching materials, conduct a three-hour Lesson Study project and write the research report for describing students' achievements. The project is done as a part of mathematics education class with six credits.

2-1. Aims and schedules on the Lesson Study project:

The Lesson Study project aimed to develop materials for giving high school students cultural awareness in mathematics, improve their attitudes and brief in mathematics by conducting lessons, and to demonstrate the educational value of the developed materials. The schedule to engage in the Lesson Study in the school year 2001 was as the following;

<u>Phase 1) Transition period</u> (almost April – June): Teacher educator (project director) explained first-year students a year plan of the project and explained what kinds of activities were expected. Second-year students in master program who engaged in last year's projects conduct new first-year students' classes to review the activities from their

actual lessons on the previous year's project. First-year students learned how to use the computers in their Lesson Study from second year students and began the project.

Phase 2) Reading of historical sources in mathematics (almost July – August): Students read historical textbooks (English readings or Japanese translations of primary sources) for excavating teaching materials and *A History in Mathematics Education* (John Fauvel, Jan Van Maanen. 2000) for learning the educational value and teaching methods of mathematics history. Teacher educator supported their reading, made clear interesting points when compared with today's mathematics and excluded the misinterpretation originated from reading mathematics history books with today's mathematics such as Bourbaki.

Phase 3) Subject matter development (almost September – November): Students developed subjects from historical texts, conceptualized lessons, established aims and goals, and developed teaching materials such as textbooks using original (or English translation) texts, slides and activities with computer. Teacher educator helped to find interesting materials from historical texts and supported students to develop structures of textbooks and lessons.

Phase 4) Lesson implementation (almost November – December): Students conducted the lesson. Teacher educator supported students to expect classroom students' activities, especially classroom students' responses and how teachers can use the response. Teacher educator also supported how to use classroom equipments such as projecting students' notebook activities to the screen for sharing students' ideas in the classroom.

Phase 5) Report preparation (almost December – February): Students wrote their research reports, created their web site. Teacher educator supported their references depending on their research problems and also supported their preparations for presentations among the mathematics education society.

IV. Analysis of the Case

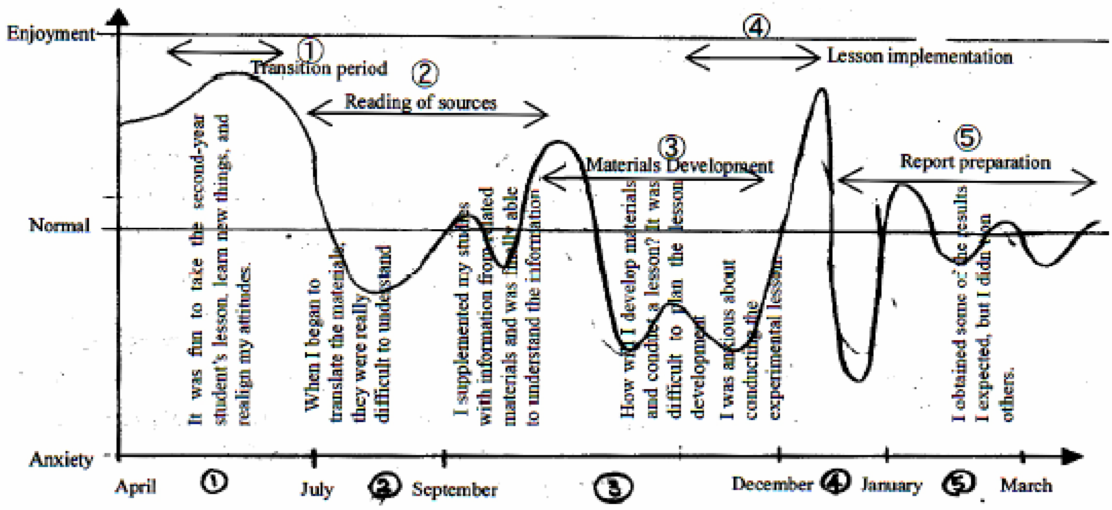
1. Analysis of the prospective teachers' experience through the project

Fourteen prospective teachers in master program participated in the project at school year 2001. After the phase 5, the researcher asked to represent how they changed through the project into the graph of emotions (see Appendix): The x axis of the graph is the time and the y axis is decided by each person, prospective teacher, for representing his/her own emotional change. Each person divided the graph by the periods for describing his/her emotional changes and the graph was explained with the periods by him/her. Thus, up and down of each graph is interpreted by each person's commentaries.

Even if each person's y axis meaning is very different, the phases are well reflected on their graphs (see Appendix: The periods ①~⑤ are rewritten in relation to Phase 1~5, not as same as original periods written by the persons.). In relation to the phases, graphs were categorized as follows: Like the graphs of Appendix 1, two persons' emotional changes are clearly related with the phases. Like the graph of Appendix 2, two persons' emotional changes did not exist phase 1 but other phases are matched with the graphs. They did not recognize phase 1 as a part of project because it was lectured by the second year students. Then, those four persons are clearly related with the phases. Like the graph of Appendix 3, three persons drew their growth of emotion and the highest emotional response is at the lesson implementation phase 4. Like the graph of Appendix 4, three persons connected Phase 2 and Phase 3 because they felt a very strong interest to read historical text as different mathematics and found their original subject matter for their Lesson Study from their readings. Like the graph of Appendix 5, two persons drew a valley at Phase 3 because they could not easily develop appropriate subject matter for teaching in classrooms. Other two persons' graphs are not clearly related with phases: One of them drew a gradual going up the graph and specially grew up at Phase 4 because he/she finally found strong mathematical interest in his lesson content. Another person drew just down after phase 1 because he/she chose the most difficult text, and felt strong difficulty in reading. He/She did not understand it well at the lesson implementation. He/She commented these kinds of mathematics are very far from school mathematics. All fourteen persons described their first impressions of projects in Phase 1 as interesting activity because they did not know school mathematics with historical text and how to use technology in mathematics. At the same time, even if teacher educator and second graders explained difficulty to read historical text and to develop subject matter from it, they could not imagine what they are and how hard they are to do.

2. An interpretation of a case

Even if we can analyze most of graphs in relation to phases, each prospective teacher's experience is very different. The explanations of periods described by each person are just their experience. Following figure is translated in English from one of Appendix 1. Handwritten numbers of ①~⑤ on the x axis are original descriptions of periods and they match to phases, clearly in this case. Here we interpret this person's emotional experience in the following way (Masami Isoda. 1998, 2000, Maitree Inprasitha, 2001): Depending on emotional theory by George Mandler (1984) based on the Piajetian cognitive model, emotional arousal is related with obstacles and challenges, and results such as overcoming obstacles give positive emotional feed backs. This cognitive cycle until reflection is also reasonable from the educational meaning of experience described by John Dewey. Based on Mandler's meaning of emotional change, we can interpret one down-up in the graph recognized as a strong experience.



Picture 1. A Case of one prospective teacher's experiences in the project

In this case, we analyze personal experience as follows: In period ①, this person (P) felt fun but did not have strong experience. P participated as a student in second year students' lessons and just enjoyed to learn last year's project. In period ②, there are two strong experiences (two down-ups). P began to read historical text and met the difficulty. P got some understanding of the text but did not understand it well. Then P found related two Japanese translation books and other supplementary books for trying to understand deeply. In period ③ and ④, there are intersections because P continued to develop materials during lesson implementation. P did not know how to develop materials from historical text but finally P developed: the strong experience of period 3. P felt anxiety to conduct the lesson but P implemented: the strong experience of period 4. In period 5, there is a deep valley after lesson implementation. It is a strong experience because P did not know how to write the report of the lesson. Next small down-up is developing the web site and P did not know the way also.

3. Didactical meaning of each phase for prospective teacher education

Even if there are two cases which did not well change the graphs in relation to phases, other twelve cases' graphs were explained in relation to phases. Their comments such as the ones seen in the case of figure 1 implicated each phase's didactical meaning for prospective teacher education. For clarifying didactical meaning of phased based on their comments, we would like to framework for interpretation of these data. Hans Nilse Jahnke (1994) used double circles for explaining historian's activity'Hermeneutics' in mathematics. First circle represents mathematician's activity on history and second circle represents historian's activity such as interpreting historical texts and asking why mathematicians did so. His model well represents the difference of mathematician's perspective and historian's perspective. Jahnke's double circles explain an activity of Phase 2. Here, we would like to expand his model to the field of teacher education for explaining nesting features of developing teachers' perspective 'kodomo womiru me' in the case of this Lesson Study project.

Figure 2-1 explains Phase I activity. Prospective teachers who are participating in the project enjoyed past project's lesson as students. They explored unknown mathematics originated from historical textbooks but reconstructed with educational questions by known mathematics.

Figure 2-2 explains phase 2 activity. They began to interprethistorical texts with known interpretations and were astonished with their differences when compared with today's mathematics. Figure 2-3 explains phase 3 activity.

Figure 2-1

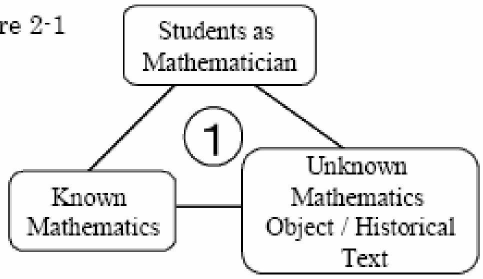


Figure 2-2

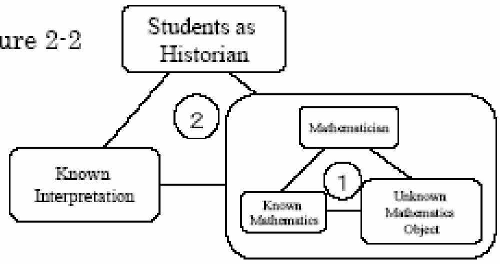


Figure 2-3

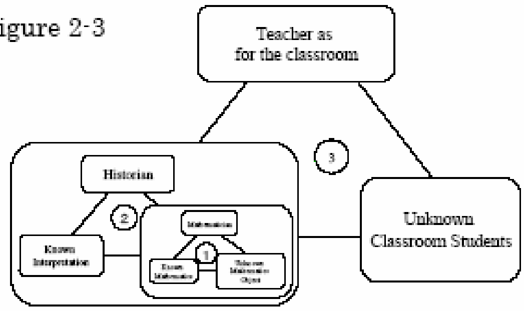


Figure 2-3*

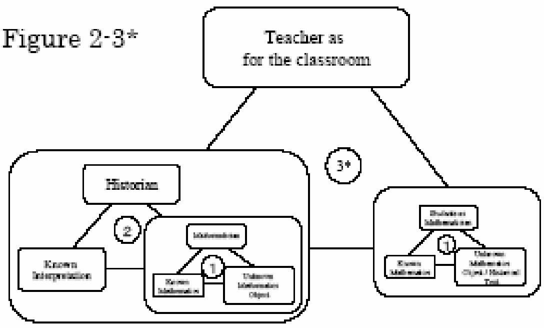
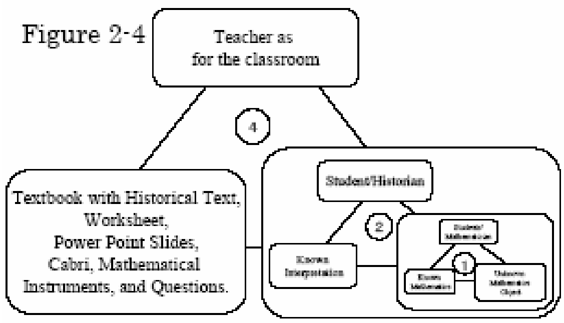
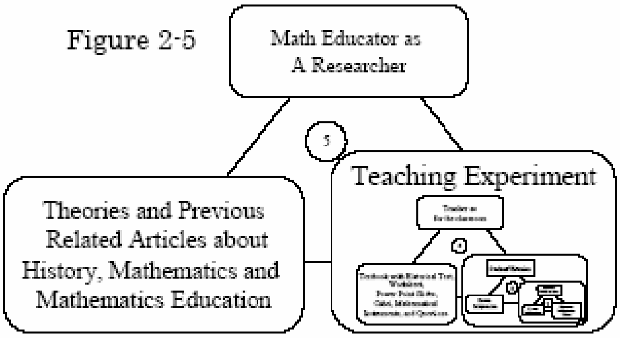


Figure 2-4



They began to develop subject Matter. Before the project, they had experience of teaching with existed textbooks and it is the first experience for them to develop the textbook of totally new subject. From historian's activity on figure 2-2, they have to develop students activities with questions for the interpretations of textbook and they have to develop their aims of their lesson study project through thinking about what students can learn from their developed activities (figure 2-3*). It is very difficult for them because of their past experience of mathematics teaching is only related with mathematical problems but in this project, they have to make historical questions at the same time. Figure 2-4 explains Phase 4 activity. Finally, they had developed materials at phase 3 and then, they tried to conduct students' activities like mathematicians and historians. Figure 2-5 explains Phase 5 activity. They reflect on both of the teaching experiment of Phase 4 and all process of the project and redefine their research questions depending on what they did and analyze it with references.

Figure 2-5



Based on the analysis, we conclude the following didactical meanings on each Phase for prospective teacher education.

<u>Didactical Meaning of Phase 1</u>: It functioned to know the activity in the lessons through enjoying lessons in past projects like students. Even if teacher educator and second graders explained what the project is and what is necessary to do, such as questionings to classroom students, students, prospective teachers, could not imagine really the meaning because they still work as students who participate in the lessons.

<u>Didactical Meaning of Phase 2</u>: It functioned to know historian's activity such as constructing the meaning through the interpretation of historical texts. Many students felt difficulty to read historical texts at first, then they were astonished with the difference between today's mathematics and historical mathematics.

<u>Didactical Meaning of Phase 3</u>: It functioned to know developing subject matter as for students' activity with historical text and technology. Some students met strong difficulties for developing classroom materials. At the beginning, many students could imagine the textbook of mathematics history and could not develop educational questions through which students can explore historical texts.

<u>Didactical Meaning of Phase 4</u>: It functioned to know conducting the lessons. Many students were scared to conduct. For knowing how to, they practiced with each other before their lessons and expected students' activity based on their questions and reactions from students.

<u>Didactical Meaning of Phase 5</u>: It functioned to know how to write the research paper based on their teaching experiments.

4. Conclusion: A nesting feature of developing teachers' perspectives

These didactical meanings with figure 2-1 to 2-5 illustrate the process how prospective teachers possibly develop teachers' perspectives in this Lesson Study project. In this project sequence, phases are constructed like nesting structures. Every teacher's education subject matter functioned to use previous experiences from different perspectives. For enhancing different meanings of perspectives, we use the word 'role' as follows.

Role of Phase 1: Like mathematician
Role of Phase 2: Like historian
Role of Phase 3: Like textbook author
Role of Phase 4: Like master teacher
Role of Phase 5: Like math-educator

We conclude that the case treated various teachers' perspectives such as mathematician, historian, textbook author, master teacher and math-educator. The sequence of Lesson Study project has nesting structures to reflect previous activity from other view points in roles. This process illustrates one of possible way to develop teachers' perspectives.
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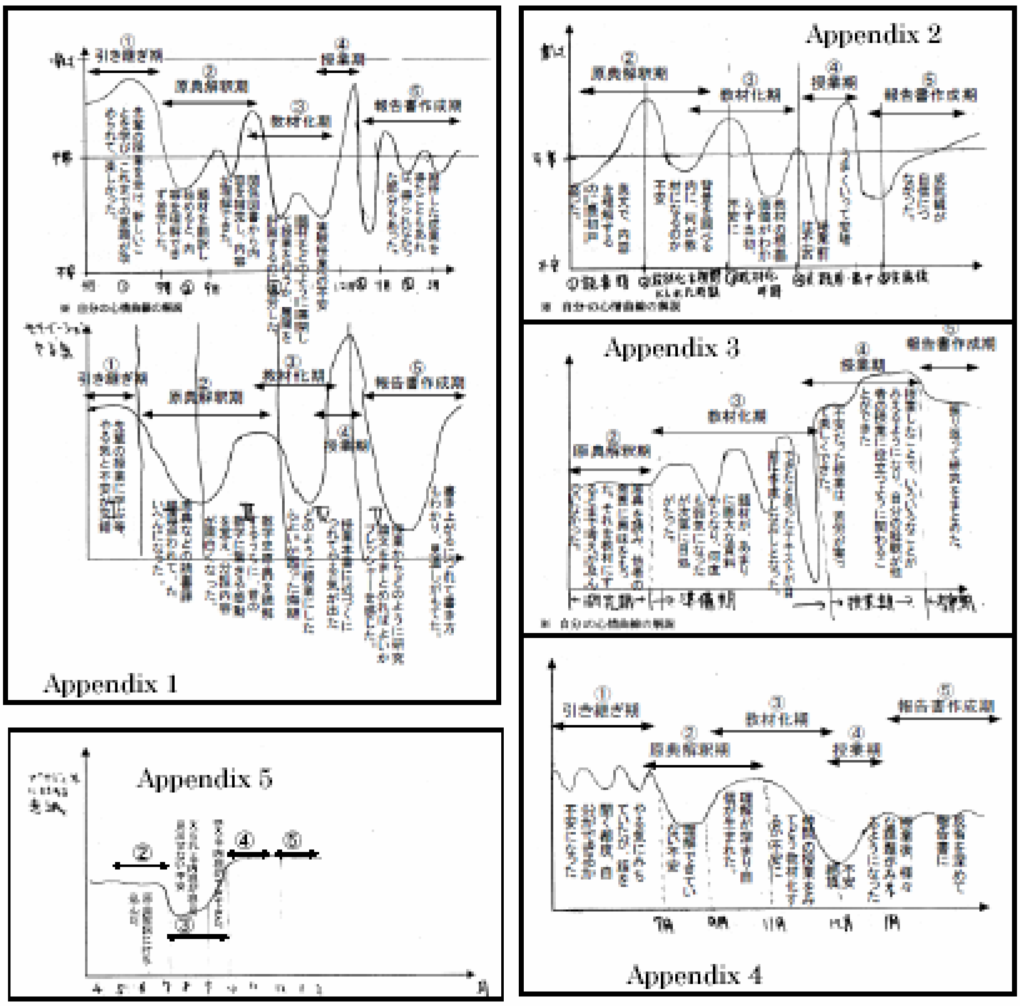
Appendix



SPECIALIST SESSION

GOOD PRACTICES OF TEACHING AND LEARNING MATHEMATICS THROUGH LESSON STUDY

FROM EQUALLY SHARING TO FRACTIONS

Dra. Grecia Gálvez P.

Ministry of Education. Chile.

This paper adresses the current situation of the Mathematical Education in Chile and a strategy developed by an University and the Ministry of Education to improve learning in the first four years of the primary school. Then a comparison is made among the version 2006 of the above mentioned strategy and the Lesson Study, as a whole-school research model. Later a Didactic Unit for the fourth year of primary school is described. This is an introductory Unit to the study of Fractions proposed by the mentioned strategy. Finally, a class corresponding to this Unit is analyzed, on the basis of its video tape recording.

Primary School and Mathematical Education in Chile.

The Chilean educational system has changed substantively since the 90's[1]. The global budget has increased significantly, as well as the wages of the teachers, the resources for learning distributed and the measures of social support to students. The infrastructure of schools has improved, the school working time has been extended and the curriculum has been modernized.

Nevertheless, the transformation of the pedagogical practices has been insufficient, with respect to what was expected from the curricular reform. There have been advances in the adoption of more active working strategies and in the incorporation of familiar contexts for the students, but it has been observed that these activities are not clearly oriented towards specific learnings, the use of the time is barely effective and the classes are weakly structured and planned. This is related to the fact that the teachers have to spend 75% of their working time in the classroom.

At the end of the fourth year of primary education all the students in the country take a test of Language, Mathematics and Science. The results of this test have not improved significantly in the last years, keeping an important gap between the performance of the children of more underprivileged sectors with respect to those that have greater economic and sociocultural resources.

Therefore, it has been considered necessary to improve the professional development of the teachers of the first primary cycle (four years), helping them to implement and to appropriate the new curriculum in mathematics and language; these areas are considered as essential to support the rest of school learning. In this context, the Ministry of Education

[1] The information outlined here is taken from: Orientaciones para el Nivel de Educación Básica 2004 - 2005, official document of the Ministry of Education.

and the University of Santiago de Chile have developed a Strategy to Support Schools in the Mathematics Curriculum Implementation. This Strategy aims improving the educational practices making workshops at each School for first cycle teachers, along with support and feedback to the educational activity in the classroom (Gálvez, 2005).

The Strategy was implemented in 20 schools (2003) and then in 224 (2004 and 2005). Since 2006 it has been redesigned as LEM Communal Workshops of Mathematics. In this modality each workshop congregates teachers from two to five schools belonging to the same commune (district), with the purpose of widening coverage to 650 schools, and it will be certified as a training activity, in order to ensure the regular attendance of the teachers. However, there is a risk of weakening the generation of institutional conditions in each school, for the installation and permanence of the changes achieved in teacher's practices.

Lesson Study and Lem Communal Workshops of Maths.

A parallel between Lesson Study (LS) in its whole-school research model version and the Strategy to Support Schools in the Mathematics Curriculum Implementation developed in Chile, in its LEM Communal Workshops of Mathematics version (LCW) is presented in the following table.

In accordance to Yoshida (2005) the steps that encompass a lesson study cycle are:	According to the Terms of Reference elaborated by the Ministry of Education of Chile (2005) LEM Communal Workshops are characterized by:
The process begins with defining a broad, school-wide research theme.	The process arises as an initiative of the Ministry of Education to improve the teacher's training in order to implement the new curriculum in the first cycle of primary education (four years).
Teachers form lesson planning teams and select a lesson study goal.	All the teachers of first cycle from two to five schools of a commune register in a Communal Workshop in which they will work during a year in Mathematics and the following one in Language, or viceversa.
The team invites an outside expert to support them.	Ministry and Universities associate to produce written and audio-visual materials and to perform assistance activities for the whole process of teaching organization in each school, through a consulting teacher, enabled by the Ministry and Universities specialists in charge of the development of the Strategy.

The team selects a unit, and within that unit, selects a lesson topic. Members of the team write a lesson plan based upon research of the topic and instructional materials.	Under the conduction of the consulting teacher, the teachers of each Workshop make weekly sessions of study of the Didactic Units produced by a Central Team. This team has selected nuclear learning from the study plan and has written four Units for each course. Each Unit is a proposal of approximately five classes, mathematically and didactically grounded, so that the teacher can lead a learning process in the classroom.
One member of the team teaches the research lesson while fellow teachers and other observers collect data on student learning and thinking.	All the teachers who participate in the Workshop put in practice the proposal contained in the Didactic Units, four times in a school's year. Some of these classes are observed by the consulting teacher or by the Technical Chief of each School (Academic Director). They can also be registered in video. The consulting teacher organizes feedback workshops (devolution), both at School and Communal level, in which the classes are commented and analysed.
The team discusses the lesson during a discussion sesion.	The authors of the Unit collect information, through the follow-in process, in order to reformulate the Didactic Units in their next versions.
The lesson is refined for the next teaching. Then the "teach - discuss - refine" cycle repeats.	Teachers who participate in the Workshop are evaluated through tests, to determine the progress of their mathematical and didactic knowledge during the year. The consulting teachers are also evaluated by means of tests but, in addition, they have to write a proposal report for teacher's training.
At year-end the lesson planning team compiles a report on the findings and outcomes of their research.	

Both LS and LCW are orientated to develop teacher knowledge across activities that lead to the improvement of teaching and learning in the classroom, to a better understanding of student thinking and to generate in teachers the need of working in a collaborative way. In LS this process is named "professional learning", whereas LCW refers to it as "professional development" or as "teacher's training".

In both models it has been difficult to explain to the administration of the educational system the principal purpose of the work that is proposed to teachers.

With regard to LS, we can mention Wang-Iverson and Yoshida (2005):

> The term lesson study, translated from the Japanese jugyokenku, has led to the mith that it means studying and improving a lesson until it is perfect (page 152).
>
> It is not easy to garner support for a long term effort designed to produce deep but incremental improvement from a district office under the pressure to rapidly raise tests scores (page 40).

In relation to LCW, a document signed by an authority of the Ministry of Education: "Unsolved Problems and Proposals in Primary Education" (Sotomayor, 2006) states:

> It is necessary to produce didactic units for the whole year, once we have the model LEM. In the course of two years the whole school year must be covered, both in language and in mathematics, from Kinder to Fourth Grade (page 2).

The promoters of both strategies, in contraposition to the mentioned statements, consider as an instance of professional learning the work that teachers make in the cycle, comprising:

- planning (with the support of the didactic units, in the case of LCW)
- implementing and observing
- discussing and reflecting (devolution, for LCW)

In relation to LS, we mention again Wang-Iverson and Yoshida (2005):

> Lesson study is the core process of professional learning that Japanese teachers use to continually improve the quality of the educational experiences they provide to their students… It played a key rol in transforming teaching from the traditional “teaching as telling” to “student cantered approach to learning” (page 3).
>
> Lesson study is a form of long-term teacher-led professional learning… and then use what they learn about student thinking and hatsumon (asking a question to stimulate student curiosity and thinking) to become more effective instructors (page 152).

With regard to LCW, in several documents in which the strategy is described we find:

> On studying the Didactic Units, to implement them and carry out its later analysis, the teachers experiment and think about their own practice, extend and deepen their own mathematical knowledge living even successive fails, they value their children’s possibilities of learning and they progress in the appropriation of a methodology to plan, to manage and to evaluate productive processes of mathematical learning. (Espinoza, 2006)
>
> Teachers use the didactic and mathematic tools acquired in the communal workshop to analyze the process (of teaching in the classroom) and the learning of the children (Espinoza, 2006).

A last dimension in which we are interested comparing LS and LCW is related to the participation of external agents in the teacher’s team.

In LS the team invites an external expert to “collaborate with them to enhance content knowledge, guide the thinking about student learning and support the team’s work” (Wang-Iverson and Yoshida, 2005, page 4). In this case, the expert provides his own theoretical frame.

In LCW we are working based on a specific theoretical approach (Chevallard, 1999). This approach considers the mathematical activity as the study of articulated problem fields. The lessons proposed in the Didactic Units are planned based on some outcome learning that have been selected from the national curriculum.

It is necessary to identify the mathematical tasks involved in these learning, which are presented to the students in the shape of problems. The techniques they will use spontaneously to explore the problematic situation are anticipated. Children will be allowed to make mistakes and stimulated to look for ways of overcoming them, on their own responsibility.

Along the sequence of classes the mathematical task, or its conditions of accomplishment, are modified in order to let the pupils experiment the need to find new techniques. By means of collective discussions they identify, among the techniques that emerge, the most effective ones. These techniques are practiced repeatedly, to generalize their appropriation in the classroom.

The problem that arises is the one of justifying the functioning of the recently adopted techniques, and then it becomes necessary to make explicit and to give a name to the underlying mathematical knowledge.

The sequence of lessons culminates with a systematization of the new knowledge, which are related to the previously acquired learning.

A Didactic Unit for the Learning of Fractions

The Didactic Unit that was used to plan the lesson that we will analyze later on was designed for the Fourth Year of the Primary School. It is called: "Comparing the results of equitable and exhaustive distributions of fragmentable objects" (Espinoza and others, 2005).

The nuclear learning of this Unit is to acquire the idea that fractions are numbers that make possible the quantification of quantities in situations in which the natural numbers turn out to be insufficient.

The purposes of this Didactic Unit are: to establish the need of the fractions as numbers, to relate the study of fractions to that of division in the field of natural numbers and to propitiate the exploration, in order to compare fractions that result from distributions of objects of the same form and size.

The chosen context is the equitable and exhaustive distribution of a set of fragmentable objects (chocolate bars) among a group of people (children). The posed problem is to quantify the part that fits to each child. In this case, the fractions emerge when the number of objects to distribute is not a multiple of the participants' number. A second problem is to compare the quantities got by each participant in two different distributions. In this case, the object of the study is the order property in the field of the fractional numbers.

The didactic strategy consists of generating a four lessons process, each lesson of 90 minutes, in which a mathematical task is proposed to the students under different conditions of accomplishment, with the aim that the sequence of situations promote the evolution of their knowledge.

The fundamental mathematical task is: to quantify the result of an equitable and exhaustive distribution of fragmentable objects. The objects are square or rectangular and they can be represented by pieces of paper of the same form.

The conditions of the distribution are:

- In the first class **1** object is distributed among **p** people, having p equal to 2; 4 or 8.
- In the second class **n** objects are distributed among **p** people, having **n < p** and p equal to the quantities of the first class, adding 3 and 6.
- In the third class **n** objects are distributed among **p** people, having **n > p** and p equal to the quantities of the second class.
- In the fourth class the relation between **n** and **p** can be anyone.

In connection to the techniques, in the first class they fragment the paper that represents the object by mean of folds and cuts and write how much each person receives, using the fractional notation. Since they only can obtain unitary fractions, a second mathematical task is proposed: to compare unitary fractions that correspond to the same object (a whole) distributed among different quantities of persons. Using techniques of visual inspection or overlapping the pieces of paper, they conclude that when the number of persons increases, the size of the part that each one receives diminishes. They deduce a criterion for the comparison of unitary fractions.

In the second class they also use the techniques of fragmenting by mean of folds and cuts but they already begin to anticipate the result of a distribution by mean of reasoning of the type: to distribute 3 objects among 4 persons every object splits in 4 equal parts and you give 1/4 to each person. Since there are 3 objects, each person will receive 1/4 + 1/4 + 1/4, it means, 3/4. This time, the task of comparing results of distributions appears as a comparison of fractions of equal numerator. For instance, the distribution of 2 chocolates among 4 persons and among 6 persons leads to the comparison of 2/4 with 2/6, which comes down to comparing 1/4 with 1/6 applying the criterion formulated in the first class.

In the third class, since n > p, we can expect that two techniques emerge:

- That they distribute complete objects first, or that they make the division n:p and, when they obtain the rest (r) lower than p, they use the techniques of the first or of the second class, according to r be 1, or more than 1. The result of the distribution will be a natural number (the whole cuocient of n:p) plus a fraction less than 1 (r/p)
- That they use the same techniques of the second class: to anticipate that it is possible to split every object in as many parts as persons there are. In this case the result of the distribution will be a fraction higher than 1, called also "improper" (n/p).

In the fourth class they will put in practice the same techniques used in the previous classes, since the tasks and its conditions of accomplishment are the same.

Analysis of an Observation of the Third Class.

The class[2] was conducted in May, 2005 by a teacher who was taking a course named "Curricular Appropriation" on Fractions, Decimals and Proportionality, in the University of Santiago de Chile. This course was given by the team of authors of the Didactic Units LEM. As a task of the course, this teacher had to design a didactic unit based on the structure of the LEM Units. Since she was working with children of fourth grade, she asked for authorization to put in practice the Unit of Fractions that we have described. Before beginning, she had several interviews with one of her teachers in order to better understand the logic of this Unit.

In the initial moment of this class the teacher illustrates the mathematical tasks that the pupils carried out in the previous two classes: share of a rectangular object among p people and of n objects among p people, being n < p. She uses folding techniques without exposing them. She emphasizes the results and the fractional notation: 1/4 and 3/4.

In the central moment the teacher proposes a distribution where n is a multiple of p. In this case, the problem is solved by a division which remainder is 0 and the result, obviously, is higher than 1.

The mathematical work of the pupils then follows. This is announced by writing the problem in the blackboard and labelling it like: "Challenge". It is a question of a distribution in which n > p and n is not a multiple of p.

The children work in teams of four. They have squares of paper, which they can manipulate in order to express their reasoning. Both the children and the teacher use only the folds, not the cuts, as they work with the papers that represent the objects that it is necessary to distribute. This can be due to the fact that the folds turn out to be sufficient to

[2] This class, observed from its record in video, is described in the Appendix.

understand the mechanics and the result of the distributions, but we also can assume a criterion of economy in the use of the material, so it can be reused.

During the sharing of ideas the teacher contrasts the results of two techniques used by the pupils where both of them are correct:

- To distribute first the whole numbers according to the model of division of natural numbers and to divide the objects corresponding to the rest, so that the distribution is exhaustive. The result is registered as a whole number plus a fractional number less than 1. To distribute the rest, if this one is 1, they use the technique used in the first class, and if it is different from 1, they use the technique corresponding to the second class.
- To divide each object in p equal parts and to assign to each person as many parts as there are objects, that is, n parts. The result is registered like n/p.

The teacher focuses the collective discussion on the question whether the results are or aren't equivalent, without addressing the techniques used by the pupils. In the case of erroneous techniques (to divide every object in n equal parts), she listens to its description but she does not comment on them.

Referring to the objects that are supposedly going to be distributed, both the teacher and the children use the attribute of "whole numbers", for they are complete, not yet fragmented. The same term is used during other moments to designate the result of a distribution as "2 wholes plus 1/4". In the latter case, the word "whole number" alludes to a property of number 2, which distinguish it from the second term of the sum, which would be a "fraction". A slide takes place between both meanings, which may facilitate the comprehension of the "whole" term as an attribute of a number, due to the analogy between "2 whole numbers" and "2 whole bars of chocolate", but later on that will be necessary to be distinguished.

As they receive a worksheet for each one, the children continue working as teams. The first task consists of a distribution of n among p, where n is a multiple of p. The division between natural numbers, as a resource to carry out this task, is considered to be learned before the study of this Unit. Nevertheless, some children who try to divide with pencil and paper don't manage to reproduce the learned skill. On the other hand, the technique of distribution of n objects among p delimited spaces used by other children, though slow and rudimentary (they distributed one by one), turns out to be successful.

The second task of the worksheet consists of a distribution of n among p, and where $n < p$. Before determining the result of the distribution, as in the previous task, the teacher asks the children to guess if the result will be more or less than 1.

During the sharing of ideas after working with the worksheet, the teacher considers the intervention of a pupil who says that in the first task it is necessary to do a division. We warn again that she emphasizes the result of the division, without addressing to the techniques used to obtain it.

Even more, having asked on the result of the second task, a pupil informs that they divided the n objects in halves, they distributed 1/2 to every p and what remained was divided in halves (1/4) and also distributed. The teacher listens attentively to this statement but she does not comment on it.

In general terms, it should be noted that during this class the teacher generates working spaces in which she allows that different techniques emerge in the hands of the pupils, but at the moment of summarizing the achievement, she focuses the discussion on the obtained results, instead of on an analysis of the used techniques.

In the closing moment, carried out in additional time corresponding to the playtime, the conclusions boil down to if the result of a distribution is more or less that 1, as n is more or less than p, leaving out other different, possible conclusions of the work made in this class.

Testimony of the Teacher that Conducted the Class.

In an interview held four months later, this teacher referred to her learning in the course of "Curricular Appropriation" and, especially, to her experience of having put in practice the Didactic Unit on fractions. We transcribe some of her statements.

> In the LEM Units the planning comes very well constructed. Nonetheless, one has to work. It is not just a matter of copy. One has to study the Unit to know what step is going to be given, what work is going to be done, and to adapt it to the reality of one’s course. The Unit of fractions helped me to raise another type of problems to my pupils. And they could solve them. The Unit served me as a guide because one can have an immense castle but if one does not work well, it could crumble down.
>
> I learned to have a clear notion of the task, the mathematical task that is going to be made by the child. When the task remains diffuse the child loses time because she or he does not know what he or she is going to do. If the teacher clearly understands the task the child does not lose time.
>
> I learned to give the children more work space during the class. I am enchanted by the way at which I work now, because the children are eager to participate. It is not important if they are wrong. If they are wrong I leave them, during a suitable time. Or they take the problem to themselves for home.
>
> I have now a passionate interest about the things that children say. With the Unit, I could work by other ways and means, and watch what happens with the pupils. The children get enthusiastic, they think. They can draw conclusions, and they feel comfortable when they do it. They go back and advance, in agreement to what they have concluded previously. They are discovering things. They value the opinion of their classmates.

> I wouldn’t be able to return and give the classes the way I did it before. They were so boring, so square. I was imposing the learning. Everything was given, was made. In fractions you had to show them the little cake, the little apple. This is a 1/2, I wrote, without opening them possibilities in order to think, to go further.
>
> The implementation of a Didactic Unit means more work. But eventually it is less work, because the children learn more. They realize by themselves that 1/2 is equal to 2/4. They like to work with the fractions, relate them to other topics. I feel that they have learned.

Conclusions

The comparative analysis between Lesson Study and LEM Communal Workshops allows concluding that both are powerful strategies to improve the educational practice and, at the same time, to generate processes of professional learning for teachers, which guarantees a higher stability of the changes achieved in their performance, with regard to other strategies.

One of the principal differences between Lesson Study and LEM Communal Workshops takes root in that Lesson Study assumes a higher degree of autonomy of the teachers’ team who work together, with regard to external experts. Thus, in the model of Lesson Study it corresponds to the teachers to choose the topic that they will work on and to plan a class. In LEM Communal Workshops the teachers receive a quite well structured proposal of planning, which corresponds to a sequence of several classes. On the basis of this proposal, the teachers organize brief processes of study that culminate with a test to evaluate what the pupils have learned.

In this paper we have shown evidence that indicates that teachers who use the LEM Didactic Units, after having studied them together with other colleagues, achieve to manage their classes in a different way from the habitual one, opening spaces in order that their pupils carry out mathematical work during the class and take part in the construction of knowledge that correspond to their study plan.

But besides, in the same amount in which the teachers appropriate the mathematical tools and didactics contained in the LEM strategy, they are acquiring a higher grade of autonomy in their daily planning work. Paradoxically, the study, application and later commentary of very specific proposals, contained in the Didactic Units, lead the teachers to advance in a process of appropriation of what is necessary to do for "not to impose the knowledge on the pupils" and for "to give them space in order that they work at the classroom, make mistakes, think and draw conclusions ", as the teacher whose class we have analyzed in this paper says.

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APPENDIX

1. Information about the VTR

Title: Equally sharing fragmentable objects

Topic: Comparison of fractions, as a result of equally sharing fragmentable objects

Producer: LEM USACH Project, 2005. Headmaster: Dra. Lorena Espinoza. Faculty of Sciences. USACH, Chile.

Context: Curricular Appropriation course on Fractions, Decimals and Proportionality. Imparted by: Dr. Joaquim Barbé, Prof. Francisco Cerda and Prof. Fanny Waisman. 2005.

Video recorder: Prof. Francisco Cerda

Video editors: Alfredo Carrasco and Francisco Cerda

Teacher: Isabel Becerra

School: Colegio Altair. Comuna Padre Hurtado. Santiago.

Grade: Fourth Year of Primary School

Date: May, 2005

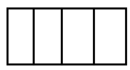
2. Description of the Observed Class.

The teacher begins, in the *initial moment,* with an inventory of the activities carried out in the previous two classes.

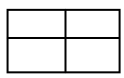
She presents 1 cardboard rectangle, she says "it is a whole" and folds it in 4 equal parts to simulate 1 chocolate that is distributed among 4 people (task of the first class). Every part is designated as 1/4.

Then, she presents 3 rectangles and folds each of them in 4 equal parts to simulate a distribution of 3 chocolates among 4 people (task of the second class). A student answers to the question about how they would make it: "I would divide each chocolate in 4 parts and I would give 3 pieces to each person". The teacher makes the folds and writes 1/4 in each part, that is to say, 4 times in each rectangle. A child writes in the blackboard the result of the distribution: 3/4.

It draws our attention the fact that she makes 3 parallel folds in the first rectangle:



On the other hand, in the other 3 rectangles she makes two perpendicular folds:



Though the rectangles are of the same form and size, nobody questions the fact that the same quantity of chocolate (1/4) is represented by not congruent surfaces.

In the *central moment* the teacher proposes a distribution of 12 chocolates among 3 friends. She writes 12:3 = 4 and she comments that each child receives 4 “full”[3] chocolates.

Then she writes a "challenge" in the blackboard:

9 chocolate bars are distributed among 4 friends

¿How much chocolate receives each one?

Children are assigned to teams of four. The teacher shares out 9 squares of paper to each group and she allows them to work freely.

We observe different techniques to accomplish the proposed task. The recording allows to distinguish the work of three groups.

Group 1. We can see a very concentrated child, with his two hands in front, moving his fingers as if he was counting them. Then he explains to his classmates: "2 for each one and the bar that remains is divided in 4 pieces" He makes two perpendicular folds in a square to obtain 4/4. He says: "each one receives 2 wholes and 1/4". Then he explains: “for you, 2, for me, 2 ... there are 8 bars. It remains 1: 1/4, 1/4...” He makes the gesture of distributing, folding the paper but without cutting it.

Group 2. A girl distributes 2 squares for each person of her group. She folds the ninth square obtaining 4 equal parts, and she simulates to distribute 1 part to each one (she doesn’t cut it).

Group 3. A girl proposes to divide each chocolate in 4 parts and to give one of these parts to each person. Thus, each person would receive 9/4 of the chocolate bar.

In this group another girl argues that each person will receive 2 bars and 1/4 of 1 bar, following the same reasoning observed in the previous groups.

In another group they fold each square to obtain 9 equal parts.

The teacher listens to the children who divided each square in 9 equal parts, but doesn’t comment on their technique.

The teacher organizes a summarizing where she confronts two techniques:

- To distribute first the whole objects and then to divide the remaining object. The result is registered in the blackboard as: 2 + 1/4.
- To divide each object in 4 equal parts and then to distribute all 36 resultant parts. The result is registered as: 9/4.

The teacher asks if it is the same thing: 2 + 1/4 and 9/4.

To show the second procedure, the teacher takes 9 squares, each one folded in 4 equal parts, and she indicates one of these parts as she counts them, to verify that they are 9/4.

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[3] In spanish, she says: “enterito”, using the same word that we use for whole number (número entero).

Some children take part to argue that it is the same thing, because with 4/4 they make 1 whole (a bar of chocolate), with 8/4 they make 2 wholes and with the last 1/4 they complete 2 wholes and 1/4. They never work with cut parts to show this equivalence.

Later they work in an individual worksheet of the Unit. The teacher allows them to continue the team work.

The first activity proposes a distribution of 42 bars of chocolate among 6 children. They have to anticipate if each child will receive more or less than a bar of chocolate and have to write with numbers the amount of chocolate each child will receive.

A few children try to make the division 42:6, but they do not remember the procedure. They say "2 in 6 fits 3 times" and they write 3. Then they say "4 in 6 fits once" and they write 1. So, they write 31. Since it seems to be too much, they invert it, leaving 13.

In another group they decide to do the distribution with objects. They put their pencils together until they have 42. They share them in 6 groups. A child says: “this way we are going to finish tomorrow!”, but the girl who is sharing continues doing it. Finally they count the pencils of each group and say: “7!”.

The children work then at another distribution of 5 objects among 6 people, with the same questions.

The teacher organizes a summary asking for the result of the first distribution. They give the answer: 7. Some children say that they have divided and others that 6 times 7 is 42. They answer that each child gets more than 1 chocolate.

As for the distribution of 5 among 6, the pupils say that each person gets less than 1 bar. A pupil explains that in his group they divided all 5 chocolates in halves, with what they would obtain 10/2. They gave a half to each of 6 persons and then they divided all 4 halves that they still had to distribute again the obtained pieces... The teacher listens but doesn’t comment on the technique that they used.

In the moment of closing, already out of the time of the class, the teacher asks them to draw conclusions:

"How much corresponds to each person if the quantity of objects to be distributed is bigger than the amount of people? More than 1 or less than 1?"The children answer: "More than 1"

"And if the amount of objects is smaller than that of people? ", the teacher asks. The children answer that less than 1.

3. A Workshop for Teachers.

1. Watch the video and comment on it freely.

2. Questioning.

   This phase deals with teachers solving problems related to the topic approached in the class and analyzing the techniques that they used and the mathematical and didactic knowledges that they have employed. If it is necessary, they complement their knowledges.

   Problem 1. In a meeting 17 people decide to order pizzas so that each person can eat 1/6 of a pizza. How many pizzas do they have to order?

   Problem 2. In another meeting 24 people order 5 pizzas of the same type of those of the previous meeting. They distribute them in an equitative and exhaustive form. Determine if in this case every person will eat more or less pizza than that in the previous meeting.

   Problem 3. Establish a sequence and explain it in order to present it to a fourth grade class, having the following tasks:

   To distribute 5 chocolates among 3 children

   To distribute 1 chocolate among 6 children

   To distribute 14 chocolates among 7 children

   To distribute 2 chocolates among 4 children

3. To watch again the video and to stop it to discuss about:

   Initial moment:

   To identify the mathematical tasks.

   To justify the equivalence between 1/4 obtained by 3 parallel folds and by two perpendicular folds in a rectangle of paper.

   Central moment:

   To identify the mathematical tasks.

   To identify the techniques used by the children to solve the problem of distribution of 9 among 4.

   To justify the equivalence between 2 + 1/4 and 9/4, and to comment on the way in which it was managed by the teacher in the observed class.

   To identify the techniques used by the children to solve the problem of distribution of 42 among 7.

To propose a reaction, on the part of the teacher, to the technique described by a pupil to distribute 5 among 6 (to divide by the half).

Closing moment:

To determine what other aspects might be included in the closing of this class.

4. To compare the comments made during the first and the second time they have seen the video.
5. To draw conclusions based upon the proposal contained in the video and upon the way in which they habitually teach this topic.
6. **Homework:** To write a paragraph on the relation that the pupils can establish between division in natural numbers and fractions, as quantification of parts of a whole object.

TEACHING AND LEARNING MATHEMATICS THROUGH LESSON STUDY – AN EXAMPLE FROM HONG KONG[1]

Frederick K.S. Leung,
The University of Hong Kong
Yuk Ying Yuen, CCC Hoh Fuk Tong College. Hong Kong

Introduction

In recent years, continuing professional development of teachers has been strongly advocated in Hong Kong (ACTEQ, 2003), and one of the means for professional development of the teachers is through lesson study, where teachers of the same school work together to study their own lessons for the improvement of classroom teaching (Fernandez. and Yoshida, 2004). Lesson study involves teachers preparing lessons together, observing and evaluating each others' lessons, and having discussions throughout the whole process. They share their teaching experiences and form a supportive group and review their classroom practice regularly.

In this paper, an example of a lesson study in Hong Kong in the subject area of mathematics will be reported. The level of students involved is grade 10, and the topic chosen for study is: ***To enhance the teaching and learning of Mathematics ---"Solving Simultaneous Equations by Graphical Method".***

In the following sections, lesson study as understood by teachers involved in this project will be discussed. This is followed by a description of the background of the study. The procedures for conducting the lesson study will then be described in detail, and the results of the study will be presented. Finally, some reflections on the lesson study will then be made, and the limitations of the study will be pointed out.

What is lesson study?

Lesson study may be considered a kind of action research consisting of a spiral of steps involving planning, fact-finding and execution. The process may be perceived as an action-reflection cycle of planning, acting, observing and reflecting (McNiff, 2002; see Figure 1 below).

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[1] Paper to be presented at the APEC International Symposium on Innovation and Good Practice for Teaching and Learning Mathematics through Lesson Study, 14-17 June 2006, Khon Kaen, Thailand.

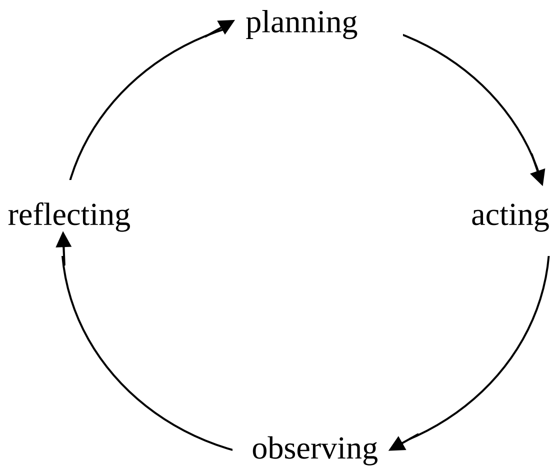

Figure1: Lesson study cycle (McNiff, 2002)

During each cycle, it is expected that small incremental improvements would be made, and as a result experiences of good practices can be accumulated and can be shared among teachers.

The first step of a lesson study is to identify a problem in teaching and learning, and this is usually achieved by a group of teachers gathering together and discussing the problems they have encountered in their past teaching experience. After identifying the problem, the group of teachers draws up a plan to collect the data they need to know about the present situation of their students, and the criteria of success for the study are established. Then the teachers prepare the lessons together, and one of the teachers teaches the lesson as planned. The teaching is observed by the rest of the teachers, and the lesson is usually videotaped. After that the teachers evaluate the lesson together to find ways to improve the lesson. The teaching plan is then modified, and the lesson is taught to another class using the refined lesson plan. Then the teachers gather together to evaluate the effects of the actions taken. Evidences are gathered to assess how far the criteria of success have been met. Finally the teachers reflect on the whole process together to identify ways the process has impacted on their work and their professional development. The impacts of the lesson study on the whole school, if any, are also discussed.

Background

The study took place in a medium sized secondary school (student population of about 1100) in a sub-urban area of Hong Kong. The school has been established for about forty-five years and is run by a Christian organization. Students are of average academic standard, and there is an emphasis on enhancement of teaching and learning by the school. The medium of instruction[2] of the school is Chinese. There is a culture of collaboration among teachers in the school, and continuing professional development of teachers is emphasized.

A group of four mathematics teachers, under the leadership of the co-author of this paper (who is also the panel chair of the mathematics department in the school), participated in the study. The four mathematics teachers (including the panel chair) involved in this project were all teaching different classes of Secondary 4 (i.e., grade 10), and the target group for study was the Secondary 4 students in the school. The aim of the study is to find ways to improve and enhance the teaching and learning of mathematics, leading to professional development of the teachers concerned. It is believed that the most effective way of improving teaching is for it to be done in the context of a lesson study. In this paper, lesson study of one lesson only will be reported.

Procedure

Timeline

In the first meeting of the project, the topic for study was decided based on an analysis of the current situation of the school (see below), and the plan and timeline for implementation were set. The timeline was established so that team members knew how the time would be spent at different stages of the project. The first meeting was held in November 2004. After negotiation among team members, it was agreed that the lesson study itself would be conducted in the last week of April 2005. This was because all team members had a lot of duties in the first term and extra time was needed to do the preparatory work. Two S.4 (i.e., grade 10) classes were chosen for this study. A pre-test would be given to the classes about one week before the lesson.

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[2] In Hong Kong, about 3/4 of the secondary schools use Chinese, the mother tongue of the vast majority of the population, as the medium of instruction (MOI), and the MOI for the remaining schools is English.

The second meeting was held in February 2005, and the main issue discussed in this meeting was about the lesson plan (see below). The third meeting was held in the first week of April, and work and duties were distributed among the teachers involved.

The fourth meeting was held immediately after the first study lesson had been conducted, and the main purpose of the meeting was to evaluate the first lesson and suggest modifications. The fifth meeting was held after the second trial, when evaluation and reflection for the whole project were conducted. The whole study finished by May 2005. A teaching assistant for mathematics in the school was responsible for preparing the videotapes for the lessons under study. The procedures above can be illustrated in Figure 2 below:

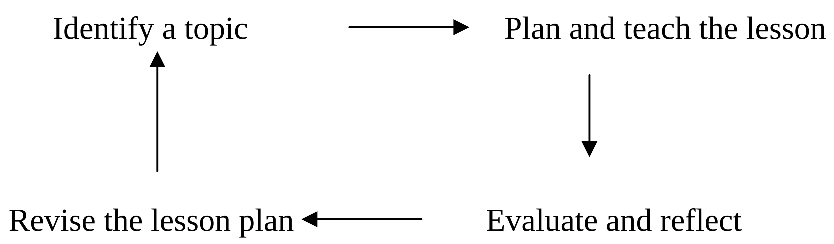

Figure 2: Procedure for conducting a lesson study

Analysis of the current situation

Before the lesson study, a holistic review of the current situation of mathematics teaching and learning was conducted. Facts on the current practices and ways of improving the practices were gathered. First the mathematics curriculum in the school was studied critically by the teachers who participated in this project to ensure that the requirements of the teaching syllabus were well understood by the participating teachers. Secondly, the study team wished to know more clearly about the standard of the students and the problems they faced in learning mathematics. Meetings were held to discuss which topic to choose for this lesson study in order to enhance the teaching and learning of mathematics. A number of mathematics topics have been considered for study, and after discussions among the team members, the topic ***"To improve students' understanding of solving simultaneous equations by graphical method"*** was chosen for study. The reason for choosing this topic was that the team found that most students were weak in plotting and reading graphs. They lacked practices in plotting graphs in their junior forms. It was hoped that through improved teaching, students' ability in handling graphs would be improved, as graphical method is important in learning mathematics and other related subjects. Relevant teaching materials and documents such as the scheme of work and the textbook were then studied carefully by the team. A pre-test was given to the students before the lesson in order to get a clear picture about whether students had a good command of the prerequisite knowledge for that topic. In order to know more about the strengths and weaknesses of the students, teachers of junior forms were consulted too.

During the meetings at this stage, the team tried to be critical of their own teaching methods and the way their students learn, and several aspects of the weaknesses in classroom teaching and ways of implementing changes were identified. The team was very clear about the teaching goals and the learning targets, and teaching strategies were designed to help students attain those learning targets.

Preparation of the lesson plan

A lot of time was spent in discussing the lesson plan. The following points were considered while preparing the lesson plan:

- to understand the prerequisite knowledge of the students
- to set clearly the teaching goals of the lesson
- to find out the misconceptions or knowledge gaps of students in learning this topic
- the use of a suitable software and good design of worksheets to help the students understand the underlying mathematical concepts and skills
- the instructional strategies and the learning activities during the lesson, that is the interaction among the teacher and the students
- the time allocation for each learning activity and the ways to assess the understanding of the students.

After discussion, it was decided that students should know first how to solve two linear equations simultaneously by graph and then used a similar method to solve one linear equation and one quadratic equation graphically. The software "Sketchpad" would be used for illustration during the lesson. The final lesson plan can be found in Appendix 1.

Modifications of the lesson

After the lesson planning, one teacher did the teaching to one of the classes (S.4C) according to the lesson plan and the lesson was videotaped in order to find out how the students learned in the classroom and their responses to the teaching. After that lesson evaluation was carried out among team members and various ways of improving the teaching practices were discussed. For example, it was found that the original plan included too much content and some parts of the lesson had to be deleted. Also the effect of the use of IT was not so good, and some modifications were done. Then the lesson plan was revised and the lesson was taught by another teacher to another class (S.4B) about one week later using the refined lesson plan. This second lesson was also videotaped for the final evaluation and also for future professional development use. So there were altogether two cycles in this study.

Criteria of success

For a lesson study, it is important to establish the criteria of success so that one knows to what extent the project has succeeded. For this lesson study, the criteria of success are summarized by in able 1 below:

Table 1: Criteria of success for the lesson study

Criteria for success	Evidence gathered
Active participation of students	Active participation of the students in class, with good responses to teacher's questions and activities were observed.
Better learning of students	The students showed improvement when comparing the result of the pre-test with that of the post-test.
Improved instructional practices of teachers, and gain in professional development	From the comments of the report on the lesson observation, the feedback from colleagues was very positive. The flow of the lessons was smooth.
Improved management skills of teachers	Feedback from colleagues on the project as a whole was encouraging. Team members were willing to try the lesson study again in the next academic year.

Collaboration among team members

As pointed out above, there were four teachers involved in the project, and each of them was responsible for different parts of the project. One teacher was responsible for writing the lesson plan after discussion of the whole team. Two teachers prepared for and did the teaching. And one teacher was responsible for designing the tests, the worksheets (see Appendices 2 and 3) and the IT teaching aids. The minutes of the meetings were taken in turn by the four teachers. A teacher assistant was responsible for making the videotapes. So it was a truly collaborative project.

The four teachers in the project worked as a team, and every team member’s ideas were respected and their opinions treasured. Team members worked collaboratively and all were empowered to do the project. All the teaching materials such as the teaching plan for the lesson, the teaching aids, the worksheets, the pre-test and the post-test, etc. were prepared by the team collaboratively.

Support from the school

In the school, this lesson study project was highly supported by the principal. Actually, a whole school approach was adopted. Besides Mathematics, lesson study was also conducted in the subjects of English and Chinese. All S.1 to S.4 (i.e., grades 7 to 10) teachers of these three “core” subjects were involved in lesson study, and a common free period was scheduled every week so that colleagues could meet and discuss about the lesson study. At the end of the term, a sharing session on these lesson studies was organized by teachers of the three subjects for all the teachers in the school. This led to both school improvement and enhancement of teachers’ profession, and a win-win situation was achieved.

Results

Scores of the pre-test and post-test

As mentioned above, a pre-test and a post-test (these two tests were identical) were administered to each of the two classes under study, and the scores of the tests are shown in Tables 2 and 3 below:

Table 2: Scores for class S.4C (lesson taught based on the original lesson plan):

Marks	Pre-test (No. of students)	Post-test (No. of students)
0-19	3	1
20-39	7	1
40-59	22	23
60 or above	4	11
Total no. of students	36	36
Full marks	64	64
Mean marks	46.7	55.8
Standard deviation	15.0	10.3
Maximum mark	64	64
Minimum mark	6	6

Table 3: Scores for class S.4B (lesson taught based on the revised lesson plan):

Marks	Pre-test (No. of students)	Post-test (No. of students)
0-19	0	0
20-39	6	2
40-59	23	19
60 or above	12	20
Total no. of students	41	41
Full marks	64	64
Mean marks	51.7	55.4
Standard deviation	10.1	12.0
Maximum mark	64	64
Minimum mark	29	31

As shown in the above tables, both classes gained good improvements after the lesson. The number of students who obtained a mark of 60 or above was significantly increased.

Evaluations by team members

Besides students showing improvements as shown by the results of the pre -test and post-test scores, below are the results of the project based on the evaluations by the team members:

- The lessons run smoothly. Most of the students concentrated in the lesson and participated actively in class. Some students who did not use to participate in class even asked questions during the lessons.
- Students could find out the answers from the graphs but they did not know how to write the answers correctly. This was revised in the second lesson.
- There were too much teaching contents to be covered. This was revised in the second lesson too.
- There should be more examples in the worksheets so that the students could follow the examples and complete the worksheet.
- More discussion could be given to the students. Interactions among the students should be encouraged.
- After the first trial, it would be better if more time was allowed to reflect and evaluate on the lesson before doing the next one. There was a lack of time to go through all the teaching materials by all teachers before the lessons.

Student responses

Some of the students were interviewed informally after the lesson and their opinions on the lessons were gathered. Most of them felt that they understood the lesson and were able to complete the worksheets given to them during the lesson. They enjoyed the lesson. So the above criteria for success were all met to a certain extent. Of course there would always be room for improvement in a lesson study.

Reflections on the lesson study

It was felt that in general the lesson study has positive effects on students, teachers, and the school as a whole.

Effects on the students and teachers

- The lesson study led to direct improvement of teaching and learning.
- The teaching materials were suitable for students in this year but might not be suitable for use again next year. However teachers' experiences and involvements in the project were valuable and could be applied to other lessons.

- There was good collaboration among the teachers in the school on this project. Team members all had equal power to discuss, and to criticize the ways of conducting the lesson. They learned more about different teaching methods from each other. Through negotiations, team members learned how to understand the perceptions of others and worked collaboratively for a good lesson.
- Evaluating and reflecting upon the lesson were very important. These might lead to improvement in the next cycle. During the process, adjustments of the lesson plan could be made from time to time.
- Teachers who took part in the lesson study would see themselves making contributions to the development of knowledge and teaching profession. This kind of classroom research could improve the teaching and help teachers in their professional development.
- Managerial skills of team members, especial those of the panel head, were improved. Members learned to share their points of views and their own teaching practices. They also learned to be respectful to others in decision making.
- Enthusiasm of the teachers towards teaching was improved, and this was important in the way to succeed.

Effects on the school

At the end of the term, there was a sharing session arranged by the school for all staff. Team members shared their experiences and their reflections on the application of the lesson study to teaching and learning. There was a great impact on the teachers. As a result, it was decided that more collaborative teaching would be done in the coming year.

Limitations of the project

Though there are many advantages in lesson study, it needs a lot of time and resources doing it. In Hong Kong the work load of teachers is so heavy that this kind of lesson study cannot be done often. Scheduling of time-table so that teachers have free common time-slots for conducting lesson study is a major challenge. Recording the lessons for later discussion is one way of meeting this challenge, but the most severe problem is that teachers do not have much time to have discussions. Notwithstanding these limitations, from the view of professional development of teachers, lesson study is still considered worth doing.

Concluding remarks

Since the society and the world are changing rapidly, we need to introduce new ideas for teaching and learning from time to time. Lesson study is very suitable to be used as a tool to fulfill this need. This research-development system is worth trying in schools. Lesson study is a self-evaluation and self-correction process. The students, the teachers and the school all gain benefits from it.

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Advisory Committee on Teacher Education and Qualifications (2003). *Towards a Learning Profession: The teacher competencies framework and the continuing professional development of teachers*. Hong Kong

Fernandez, C. and Yoshida, M. (2004). *Lesson Study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, N.J. : Lawrence Erlbaum Associates.

McNiff, J., Lomax, P. & Whitehead, J. (1996). *You and Your Action Research Project.* London: Hyde Publications (pp 29-45).

<table>
<tr><th colspan="5">Appendix 1</th></tr>
<tr><td>Unit</td><td colspan="4">Graphical Solutions of Simultaneous Equations</td></tr>
<tr><td>Reference of teaching materials</td><td colspan="4">“New Progress in Certificate Mathematics”
Hong Kong Educational publishing Co.</td></tr>
<tr><td>Teachers</td><td>MFC
SKP</td><td>Classes taught</td><td>S.4</td><td>No. of Students：
36(4C), 41(4B)</td></tr>
<tr><td>Dates of teaching</td><td colspan="2">21-4-2005 (MFC S.4C)
27-4-2005 (SKP S.4B)</td><td>Teaching period</td><td>1 period (40 mins.)</td></tr>
<tr><td>Students’ Prerequisite Knowledge</td><td colspan="4">1. Understand how to plot points in a coordinate plane
2. Know how to plot a linear equation on a graph paper
3. Understand that the graph of a linear equation in two variables is a straight line
4. Understand that all the points on the straight line can satisfy the linear equation
5. Know how to use graphical method to solve a system of two linear equations
6. Understand that the solutions found from the graph are only approximate solutions
7. Understand that the point of intersection of two straight lines can satisfy both equations and hence it is the solution of the simultaneous</td></tr>
</table>

Teaching Goals	1. Students are able to use graphical method to solve a system of two linear equations of two variables.(as a revision) 2. Students are able to know that the solutions found from the graph are only approximate solutions.(as a revision) 3. Students are able to draw a straight line by plotting three points 4. Students are able to understand how to find the solution of a system of equations, one linear and one quadratic graphically and its meaning. 5. Students are able to read and write the solutions from different scales of graphs.

Preparations before the lesson:

1. The software "Sketchpad" will be used to prepare the graphs for illustration in the lesson.
2. The worksheets will be prepared. All the quadratic graphs will be provided and the students need to draw straight line graphs only

Teaching aids: Computer for demonstration and the software "Sketchpad"

Teacher's Activities	Students' Activities	Assessments and Points to be noted
Explain briefly about the learning goals of the lesson	Listen	Time:15 mins.
Explain how to use graphical method to solve a system of two linear equations of two variables using Sketchpad. Ask students to find the solution by themselves and write it down on the worksheet.	Students try to find the solution And write it down.	Check that students can find the point of intersection of the two lines and write the correct solution.
Discuss with the students how to find out the answer and explain why the point of intersection of two linear equations is the solution.	Students discuss in groups and share their opinions.	the point of intersection of the two lines can satisfy both equations
Ask the students to check the answer.	Do the computation by the aid of calculator.	By checking students are able to understand that the answer is an approximate solution only

Tell the students first that the graph of a quadratic equation is a parabola. Give an example: How to find the solution of a system of equations, one linear and one quadratic graphically. Given a graph of a quadratic Equation. Ask the students to complete the table given and use the three points to plot a straight line. Using the graph plotted by Sketchpad Discuss with the students how to find the solution of a system of equations, one linear and one quadratic. 7. Ask the students to do more examples, using graphs of different scales. 8. Ask the students what is the maximum number of solutions for simultaneous equations, one linear and the other quadratic. 9. Ask the students the reason why the point of intersection of the graphs is the solution.	Do the computation and draw graph. Discuss in groups, find the solution and write it on the worksheet. Practice more to find the solutions from the graphs Students answer “Two” Group Discussion	Time: 20 minutes See if the students can draw the graph correctly. See if the students are aware that the solution is an approximate solution. See if the students know there are different cases for the number of solutions The solution is only an approximate value by checking. The point of intersection can satisfy both equations. Hence it is the solution of the simultaneous equations
10. $\boxed{\text{Conclusion}}$ (a) We can find the solution of a system of equations, one linear and one quadratic graphically. (b) The solutions obtained from the graphs are only approximate values (c) The point of intersection can satisfy both equations. Hence it is the solution of the simultaneous equations 11. Assignment given	Listen	Time:5 mins

Appendix 2

Form 4 Mathematics

Solving Simultaneous Equations by Graphical Method (Worksheets)

Name: ____________________ Class: __________ Class Number: __________

1. Solve the following simultaneous equations by graphical method.

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

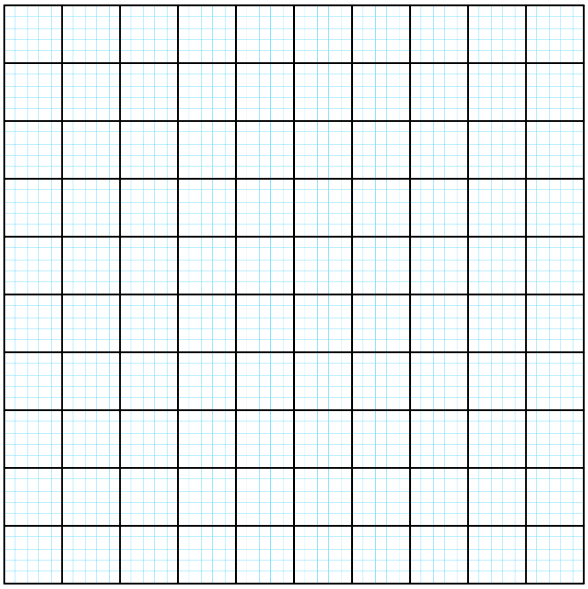
Solution

$x + y = 4$

x	0	2	4
y			

$x - y = 2$

x			
y			



$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

Checking:

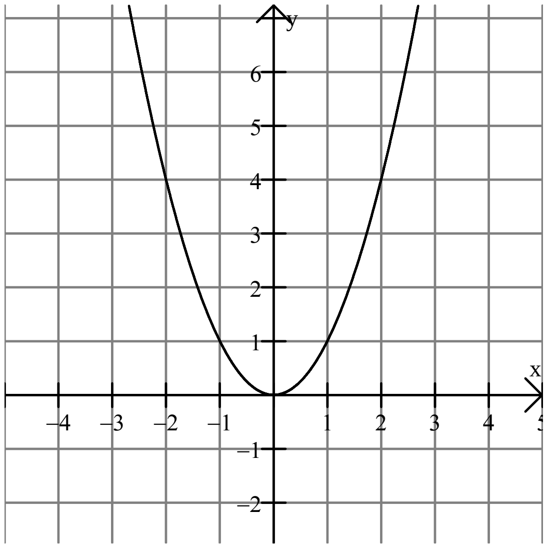
2 Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = -x + 2 \end{cases}$$

Solution

$y = -x + 2$

x	-3	0	3
y			



$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

3. Textbook p.57 (Follow-up Exercise)

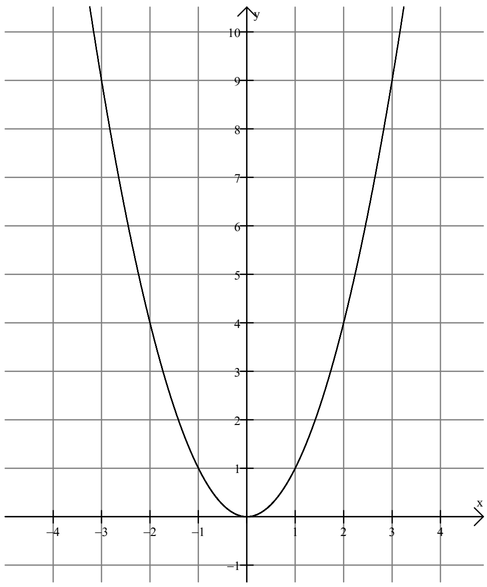
4. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = 6 - x \end{cases}$$

Solution:

$y = 6 - x$

x			
y			

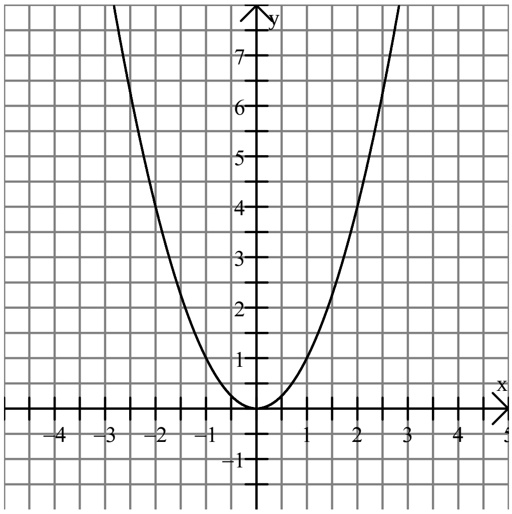


$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

4. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ 2y = 3x + 2 \end{cases}$$



Solution

$2y = 3x + 2$

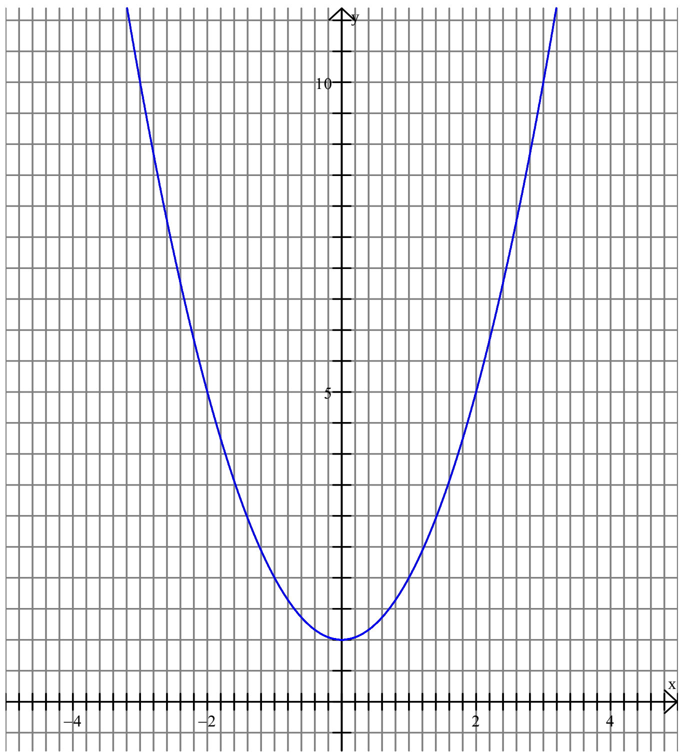
x			
y			

$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

6. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 + 1 \\ y = x + 7 \end{cases}$$



Solution

$y = x + 7$

x			
y			

$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

7. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = -x^2 - x + 2 \\ y = x - 3 \end{cases}$$

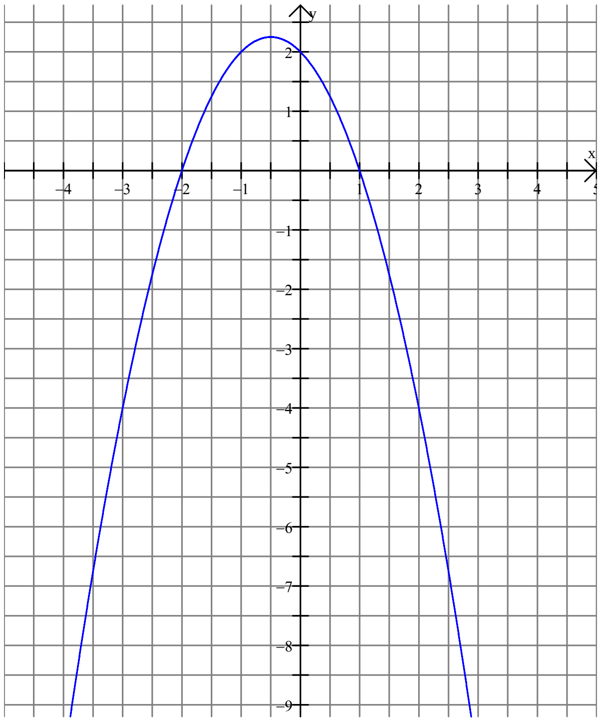
Solution

$y = x - 3$

x			
y			

$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________



8. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = 8x - 16 \end{cases}$$

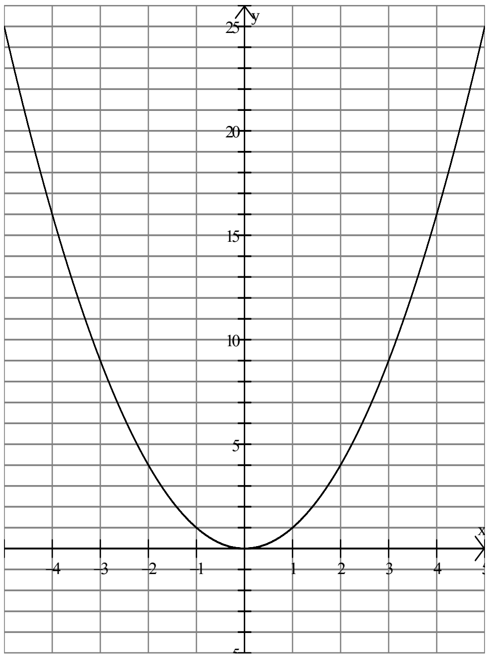
Solution

$y = 8x - 16$

x			
y			

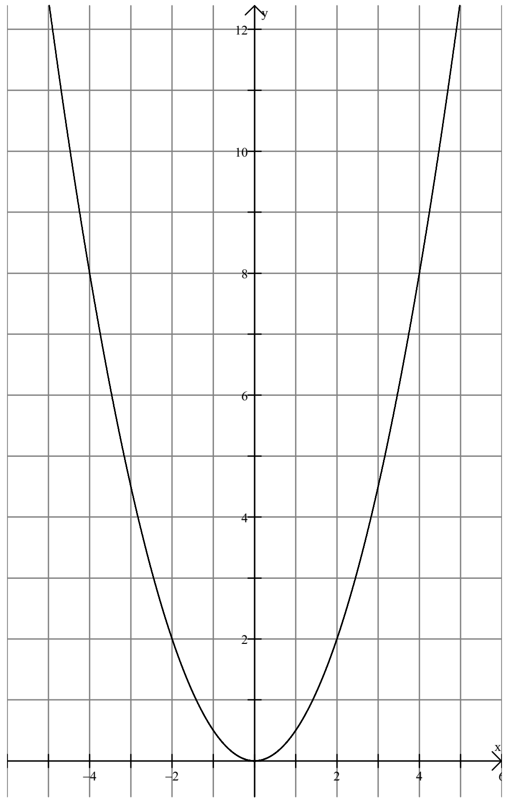
$\therefore$ From the figure, the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________



9. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = \dfrac{1}{2}x^2 \\ y = \dfrac{3x}{2} - 2 \end{cases}$$



Solution

$$y = \frac{3x}{2} - 2$$

x			
y			

$\therefore$ From the figure, we know the point of intersection is ____________

$\therefore$ The solution of these simultaneous equations is ____________

Appendix 3

Mathematics Test

Coordinates and Solving Linear Equations in Two Unknowns by Graphical Method

Name: ________________( ) Class:________________ Score : ___________

Date:________________________ Total: 64

Time: 35 minutes

1. Write down the coordinates of points A to F in the rectangular coordinate plane shown in the diagram.

The coordinates of *A* are = ________________

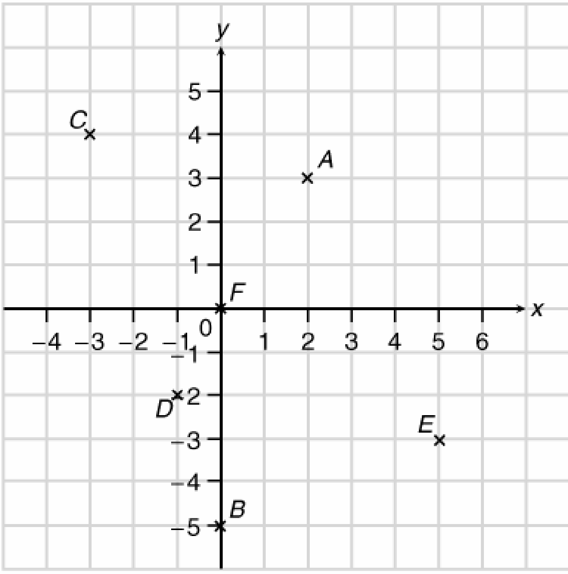
The coordinates of *B* are = ________________

The coordinates of *C* are = ________________

The coordinates of *D* are = ________________

The coordinates of *E* are = ________________

The coordinates of *F* are = ________________



(6 marks)

2. With reference to the given linear equations in two unknowns, complete the corresponding tables respectively.

(a) $y = -5x$

x	–1		
y		10	–20

(b) $3x + y = 5$

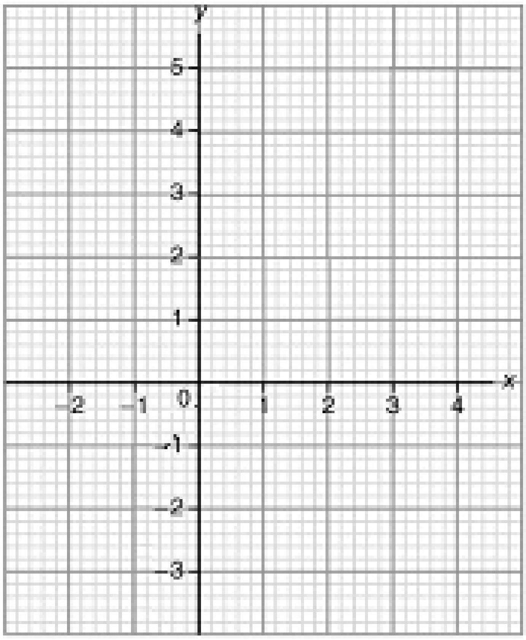
x	–2		2
y		0	

(12marks)

3. Complete the following table, and draw the graph representing the linear equation in two unknowns $4x - 3y = 1$.

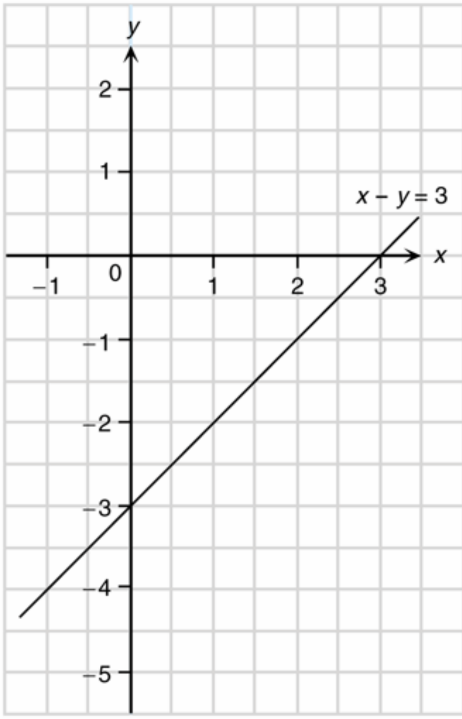
Solution：

x	-2	1	4
y			



(8marks)

4. The given figure shows the graph of the linear equation in two unknowns $x - y = 3$.



(a) **(i)** With reference to the linear equation in two unknowns $3x + y = 1$, complete the following table.

x	0	1	2
y			

(ii) Try to draw the graph representing the linear equation in two unknowns $3x + y = 1$ in the figure on the right.

(b) Use the results of (a) and graphical method to solve

$$\begin{cases} x - y = 3 \\ 3x + y = 1 \end{cases}.$$

(18 marks)

Solution：

5. Solve the following pair of simultaneous linear equations in 2 unknowns graphically.

$$\begin{cases} 2x+3y=5 \\ 3x-y=2 \end{cases}$$

(20 marks)

Solution：

With reference to the linear equation in two unknowns $2x+3y=5$, complete the following table.

x	-2	1	4
y			

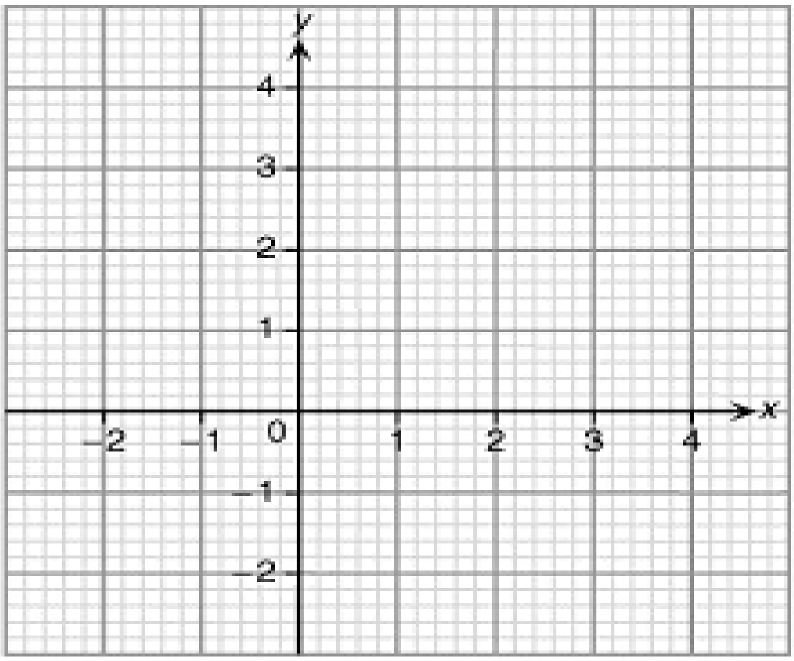
With reference to the linear equation in two unknowns $3x-y=2$, complete the following table.

x	0	1	2
y			

Draw the graph representing the pair of linear equations in two unknowns $2x+3y=5$ and $3x-y=2$ in the diagram below.

From the graph, the two straight lines intersect at the point (,).

∴ The solution of the system of linear equations in two unknowns is ______________________



End

PROMOTING LESSON STUDY AS ONE OF THE WAYS FOR MATHEMATICS TEACHERS PROFESSIONAL DEVELOPMENT IN INDONESIA:
The Reflection on Japanese Good Practice of Mathematics Teaching Through VTR

Marsigit
Faculty of Mathematics and Science, the State University of Yogyakarta, Indonesia
E-mail: marsigitina@yahoo.com

Reflecting good teaching practice of mathematics form different context through VTR was proved to evidently encourage and motivate teachers to improve their teaching competencies. In some parts of the activities of teachers professional development programs in Indonesia, year 2002- 200, the reflections through VTR of Japanesse context of teching practice (Teacher: SAITO, Kazuya; School: Ookayama Elementary School, Yokohama city, Unit: The area of plane figures) resulting teachers' perception that it was a good model of mathematics teaching that can possibly be implemented in Indonesia. However, the techers were aware that to implement such good model there are some fundamental constraints should be overcome.

Overview

VTR (Video Tape Recorder) for teacher education and reform movement in Mathematics Education, specifically for developing lesson study has some benefits as: a) short summary of the lesson with emphasis on major problems in the lesson, b) components of the lesson and main events in the class, and, c) possible issues for discussion and reflection with teachers observing the lesson (Isoda, M., 2006). According to him, a Lesson Study is divided into three parts: a) planning the lesson, b) the observation part, and, c) the discussion and reflection part.

Further he stated that when we use the VTR, we also begin from the lesson observation but the VTR itself already loses many dimensions, parameters and context because the program is prepared (recorded) from the perspective of the recorder's and VTR editor's eyes only. Through the observation of the VTR, we learn things and apply these in the next activity. Teachers in Indonesia can observe the lesson of different context in different country (e.g. Japan) through VTR.

If we observe teaching learning processes through VTR, a short summary is necessary to grasp the contents and we need to observe the VTR several times to understand its contents clearly. Having done this, it may arrise the usefull issues for discussion and reflection as well as to reflect on good practices, good lessons or innovative lessons for the reform of mathematics education.

> In the process of pre-service teacher education, it is important to develop teacher's perspectives. Learning to listen is a key word for this approach. In the case of Japan, lesson study usually begins by developing a lesson plan. At this stage, teachers solve and pose problems from students' perspectives. By analyzing problems, teachers develop good ways of questioning. For writing the description of the VTR, it is very important to ask why? Why did students say this? Behind their words, there must be so many kinds of ideas. Why did the teacher say that? Through these questions, we can better know and understand the hidden features of the lessons being observed through VTR. Then, it is very important to add the format such kinds of descriptions from the view points of original lessons but even if we add descriptions we do not needs to follow because re-contextualization is done by VTR users. (Isoda, M., 2006)

Teachers Professional Development

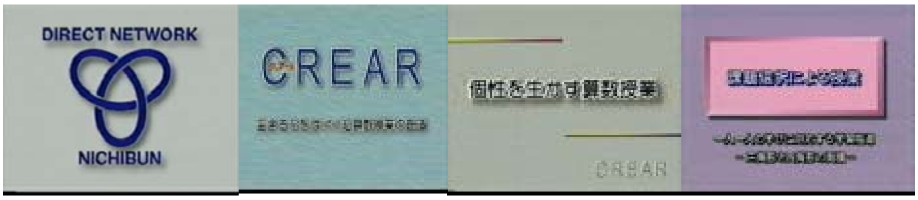
Since the early of 2000, there are cooperations among universities, teacher training institutes and MoNE's Directorate of Secondary Education to improve teachers' competencies to support the implementation of the proposed competent-based curriculum (Curriculum 2004). The author has involved in some professional development activities (workshops) surrounding Indonesia such as:

1. Validation and Socialization of the Guideline of Syllabi and Evaluation System of Competent-Based Curriculum for Mathematics in Manado, North Sulawesi. 16-17 September 2002 (40 participants from Local District)
2. National Semiloka for Socialization the Development of Competent Based Curriculum for Junior High School Mathematics in Yogyakarta, 20-25 and 27-31 October 2002 (120 participants consists of four representatives from each District)
3. Validation and Socialization of the Guideline of Syllabi and Evaluation System of Competent-Based Curriculum for Mathematics, Yogyakarta, 22 November 2002 (60 participants consists of 2 representatives from each District)
4. National Level of Training of Trainer (TOT) for Basic Science, in Yogyakarta, 4-14 June 2003 (60 participants consists of 2 representatives from each District)
5. National Level of Training of Trainer (TOT) for Basic Science, in Yogyakarta, 15-20 December 2003 (60 participants consists of 2 representatives from each District)
6. Monitoring and Evaluation of the Piloting of Competent-Based Curriculum for Mathemtics in State Junior High School I and III, Binjai, North Sumatra. Desember 2004 (20 participants from 2 schools)
7. Monitoring and Evaluation of the Piloting of Competent-Based Curriculum for Mathematics in Padang, West Sumatra, January 2005 (80 participants from Local District)

At the beginning of each of those activities, the author played the Japanesse VTR of Lesson Study produced by "CREAR" of DIRECT NETWORK NICHIBUN, to reflect teachers' perceptions and to understand the extent it influences teachers' following activities.

Picture: National Level of Training of Trainer (TOT) for Basic Science,
in Yogyakarta, 4-14 June 2003

Reflection on Japanese Good Practice of Mathematics Teaching through VTR



Lesson : Choosing Tasks according to Pupil's Interests (4th grade)
Teacher : SAITO, Kazuya
School : Ookayama Elementary School, Yokohama city
Unit : The area of plane figures
Method : Tasks based on pupils' interests.

The objectives:

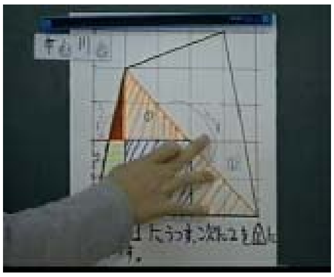
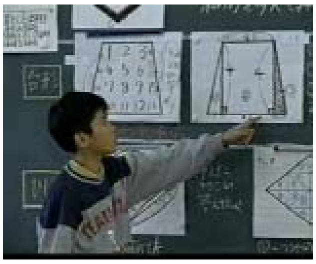
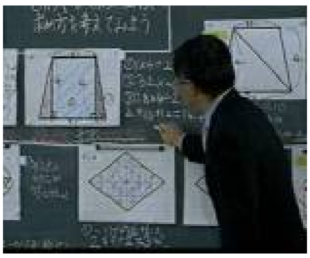
- Pupils appreciate the formulas for the area of figures and are willing to use the formulas in order to find the area.
- Pupils are able to find the area making the best use of their prior knowledge and experience.
- Pupils are also able to formulate the methods to find the area of parallelograms.
- Pupils can find the area of fundamental Figures efficiently.
- Pupils understand the methods to find the area of fundamental figures.

Highlighting the VTR:

	Teacher posed the problems as follows: Can you find the area of the figure? If you think about how this figure is made?

	Pupils begin by reflecting on their prior knowledge.' experience, and the opportunity to learn about area in the previous grade. Pupils make a plan how to decompose the figure in the problem into square, rectangle, right-angled triangles, triangles, parallelogram, or trapezoid, in order to find the area of the figure.

	Pupils realize that they have to begin by learning how to find the area of triangles. Pupils discuss how to decompose the figure. Then, they do the problem separately in three groups as follows: • The figure in the problem can be decomposed into right-angled triangles and rectangle. • The figure in the problem can be decomposed into a few' triangles. • The figure in the problem can be decomposed into triangles and parallelogram, or trapezoid.

	Pupils formulate the methods to find the area of scalene triangles Pupils try to find the area of quadrilaterals using the formula for the area of triangles. Pupils shall find the area of quadrilaterals using
	Pupils use parallelogram to find the two congruent triangles area of rectangles, and by a diagonal line, pupils shall solve the problem.
	Pupils try to find the area of parallelograms.
	Teacher encouraged the pupils to consider how to find the area of rhombus an trapezoid.

THE EXTENT THE INDONESIAN TEACHERS LEARN AND IMPLEMENT THE ASPECTS OF JAPANESE GOOD PRACTICE OF MATHEMATICS TEACHING

From the seven activities of workshops, there are totally 440 participants who observed the VTR and gave the inputs.

In each of those workshops, there are some steps of reflecting those teaching:

a) Firstly, observing the VTR without any comment from the trainer
b) Secondly, collecting the general comments from the audiences
c) Thirdly, repeating the observation of the VTR with some comments from the trainer
d) Fouthly, discussing the more specific aspects of the teaching

Picture: Monitoring and Evaluation of the Piloting of Competent-Based Curriculum for Mathematics in Padang, West Sumatra, January 2005

The following are their perceptions:

Teacher perceptions of the teaching in the VTR:

1. 100 % of the total numbers of participants perceived that the teaching reflected in the VTR was a good model of teaching mathematics.
2. 80 % of the total numbers of participants perceived that the teaching in the VTR is a good model and it needs to be socialized to other teachers.
3. 73,3 % of the total numbers of participants perceived that they are willing to discussed it to their colleagues after the training.
4. 95 % of the total numbers of participants perceived that the teaching in the VTR is a good model but there are still some constraints to implement it.
5. 53,3 % of the total numbers of participants perceived that the constraint to implement this good model of teaching is that the teachers' lack of time.
6. 33,3 % of the total numbers of participants perceived that the constraint to implement this good model of teaching is the unreadiness of the students
7. 26,67 % of the total numbers of participants perceived that the constraint to implement this good model of teaching is the limit of budget
8. 47 % of the total numbers of participants perceived that the constraint to implement this good model of teaching is lack of educational facilities.
9. 25,6 % of the total numbers of participants perceived that they were optimistically able to implement this good model of teaching by additionl time of teaching and developing lesson preparation.
10. 42 % of the total numbers of participants perceived that to implement this good model of teaching, they need to improve their competencies of teaching contents.

Teachers' perceptions of the actions following up the training:

1. 80 % of the total numbers of participants perceived that they will discuss the VTR with their colleagues
2. 60 % of the total numbers of participants perceived that they will disseminate the results to other teachers
3. 40 % of the total numbers of participants perceived that they will discuss the VTR in the teachers club
4. 55 % of the total numbers of participants perceived that they will try to improve their teaching covers: improving Lesson Preparation, Student Work Sheet, teaching content and teaching methodology.

Teachers' perception of the kind of teaching method they will develop after the training:

1. 6,7 % of the total numbers of participants perceived that they will develop Realistic Mathematics Education and Constructivis approach.
2. 38 % of the total numbers of participants perceived that they will develop discussion and demonstration methods.

3. 17 % of the total numbers of participants perceived that they will develop various methods.
4. 33,3 % of the total numbers of participants did not indicate any method.

Concluding Remark

In general, the activities of reflecting Japanesse context of mathematics teaching through VTR in the training program were perceived as good and useful by the teachers. The teachers perceived that such activities need to be socialized to other districts in order that more teachers can learn it. They perceived that the teaching reflected in the VTR was a good model that can also be implemented in Indonesian context. However, they perceived that it is not easy to implement it.

The teachers viewed that to implement good model of mathematics teaching, as it reflected in the VTR, there are some constraints coming from: lesson plans, students' worksheets, teachers' competencies, students' readiness, educational facilities and equipments, teaching methodologies, allocation of time, number of students and budgeting. Teachers need to improve their competencies of teaching and competencies of teaching contents. They perceived that they need to improve their competencies in preparing the lesson plans and producing students' worksheets.

According to teachers, most of the students are not ready or not able to present their ideas; it takes time for them to accusstomed to do that. Most of the schools are lack of educational facilities and teachers need to be able to develop teaching media. The most difficult one to implement such good model of teaching practice is about time allocation. Some teachers perceived that it is not easy to take in balance between achieving students' competencies and considering their processes of learning. Meanwhile, a teacher still should facilitate a lot number of student i.e forty students per class.

The teachers hoped that the schools and government support their professional development including the chance to get training, to participate the conferences, to participate in teachers club. The teachers perceived that in the teachers' club they are able to discuss and develop lesson plan and students worksheet. Teachers suggested that teachers' professional development programs should be based on teachers' need; and therefore, it needs such a need assessment prior the programs. They also hoped that the schools and government procure educational facilities and improve their salary.

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IMPROVING THE QUALITY OF SECONDARY MATHEMATICS TEACHING THROUGH LESSON STUDY IN YOGYAKARTA, INDONESIA

Sukirman

Faculty of Mathematics and Science, University of Yogyakarta, Indonesia

E-mail: sukirman_mipa@yahoo.com

There are many ways to improve the quality of mathematics teaching in Indonesia. Evidence and experiences from other country indicated that Lesson Study can be the one of the method for supporting teachers' professional development. From 2004 to 2005, in collaborations with IMSTEP-JICA Project, the Faculty of Mathematics and Science, the State University of Yogyakarta introduced and implemented lesson studies activities in two district Sleman and Bantul.The results of lesson studies activities indicated that there were significantly improvement of teaching learning of secondary mathematics in term of teachers' competencies and students' motivation. Amongst the success results there were also difficulties how to implement lesson studies continually in term of budgeting.

Introduction

There are many factors influencing the quality of mathematics teaching at schools such as teacher, learner, facilities, laboratory completed with experiments kits, environment, management, and so on. In this paper, it is focused on teacher as the most important factor in influencing the improvement of teaching quality instead of other factors mentioned. In Indonesia, in term of teachers' professional development, the improvement of teaching quality have caried out by various programs such as in-service training such as equivalency, training, seminar or workshop, and some the like. After completing the training, teachers are expected to implement it in teaching their students in class.

For a certain teacher training that has been conducted by Indonesian government need a large amount of budget which was taken from national budget or international loan. There was adequate feedback resulted from those trainings toward the improvement of teaching quality. Following the programs for teachers' professionalism development, they need monitoring activities to picture the impacts of the improvement on teaching quality. Program for in-service teacher training has its aim to improve the teacher quality, however, it was difficult to serve the trainee to have the chance to get "concrete experience of teaching" in training activities. There were evidences that the trainees sozialize their results of training in teachers' club.

Since 2002, Indonesian education system is more decentralized rather than centralized. It makes the schools and the teachers to have the new chalenges how to improve their quality of teaching otonomously. Schools and teachers are now to have the chance to develop their own curriculum with a few and flexible guideline from the central government. Currently, national curriculum was simply developed containing the outline of competency standard, basic competency, and achievement indicator. Teachers have their right to describe it into detailed syllabi based on students' characteristics, school resources, and environment.

In decentralization era, teachers have to be more active and creative to create and develop their ideas without unintended interventions from central government. Teachers are now to have a chance to deconstruct their old paradigm of teaching. They are not just as the implementer of curriculum but they tend to be developer of curriculum. It is not caused by bad input outside but teachers will have a freedom to explore their role professionally in class. Teachers are challenged to have trained competency to prove their profession as professional teachers. Briefly, now, teachers are the implementer of what has been decided by bureaucrats as well as challenged to think logical, critical, creative, and doing reflection in improving teaching quality. However, the central government still has their important role in facilitating teachers' professional development. One of the way to support the teachers is to introduce lesson study program to improve the quality of their teaching.

Lesson Study

Japanesse experts indicated that Lesson study is considered as: 1. intiative of a teacher or group of teachers to improve themselves in teaching, and to get any input to make innovation based on the result of good plan and implementation (open for other teachers/observers to visit their class); 2. medium for learning of teacher or other participant including the teacher as presenter; 3. medium for discussion or sharing experience to improve teaching quality.

Meanwhile, we define Lesson Study as an activity carried out by a number of teachers of a certain subjects in collaboration with educational experts to improve their quality of teaching. Lesson Study has three (step) main activities : planning, implementing (teaching & observing), and reflecting toward the planning and implementing to real teaching.

1. Planning

In this step, there is an identification of the problems found in classroom used for lesson study followed with its alternative solutions. The identification and solution taken is related to the teaching material talked in classroom, schedule, students' characteristic, class condition, teaching method, teaching media, experiment kits, and evaluation toward the teaching process and result.

There was discussion about the choosing of teaching material, method, and media based on students' characteristic and evaluation types used. There would be suggestion/input coming from teachers and experts. Experts or senior teachers would gave any opinion about new things to be applied by teachers in classroom including using the teaching approach of constructivism, contextual teaching and learning, life skill, Realistic Mathematics Education, carrying out newest teaching material or others can be brought into discussion.

Other discussion is about the writing of observation sheet especially about determining the indicator of good teaching-learning process seen from the aspect of teacher and students. Those indicators were written based on lesson plan taken and basic competency to reach out by students during the teaching-learning process.

Based on the identification and solution of the problems above, it was carried out into a set of teaching unit consisting of:

a. Lesson Plan
b. Teaching Guide
c. Students' worksheet
d. Teaching media
e. Evaluation sheet of teaching process and result
f. Observation sheet

Lesson plan can be written by one teacher or more who agreed with the aspects of planned teaching. To be more perfect, the result is, then, discussed with other teachers and experts of their group.

2. Implementation and observation

In this phase, a teacher implemented the lesson plan developed while other teachers and expert observed the process using observation sheet prepared. To support it, the observer took video shooting to take the special events during the implementation both teacher and students. There are some phases and in each phase there are some cycles of Lesson Study activities.

3. Reflection

In this phase, the teacher implemented the lesson plan was given a time to state his feeling during the implementation both for himself of his students. Next time was given to observers both expert and other teachers to give their analysis of observation data toward the students' activity during the implementation followed by the play of video. The teacher of presentation, then, was asked to respond the observer's comment. The important thing in reflection is to reconsider the lesson plan developed as the basic to make improvement to next teaching.

Is the lesson plan fit and able to improve students' activeness in learning? If not fit yet, find those are not fit. Is that about teaching method, student's worksheet, media or other teaching aids? This consideration is taken as input for improving the teaching in next phase. Seeing the aspect of planning, implementing, and reflecting on lesson study, it makes lesson study looks similar to Classroom Action Research (CAR).

Methods

In cooperation with IMSTEP-JICA Project, in Yogyakarta, Lesson Studies activities were carried out in some schools that we called piloting schools. In the 1st phase, starting in the year of 2004 in the district of Sleman, Yogyakarta, the activities of lesson studies were already conducted by some mathematics teachers from 21 secondary schools. The school selection was taken under the aspect of school representation among senior and junior high schools in villages and towns in each regency of Yogyakarta province and headmaster supported much. In conducting lesson studies we also involve the role of teachers club. There are 3 cycles of activities in the 1st phase of lesson studies.

The results of piloting program in 1st phase were enhanced in 2nd phase of Lesson Studies activities. In the 2nd phase, starting in the year of 2005, still in the district of Sleman, Yogyakarta, Lesson Studies were carried out in 42 schools (as the extension from schools numbers in the 1st phase). The use of many schools was aimed to disseminate the results of lesson studies activities to other teachers from other schools. However, because of the limitation of the budget, in the next phase, we should decrease the number of schools that is we concentrate to carry out lesson studies activities in 3 junior high schools and 3 senior high schools. In each lesson studies activities, there are 5-6 teachers in collaborations with university lectures and Japanese experts to carry out the steps of activities. The following phase of lesson studies activities is the results of the previous reflection and the results of improvement based on the inputs from teachers, lecturers and experts. There are 3 cycles of activities in the 2nd phase.

In the 3rd phase, starting in the year of 2005, lesson studies activities were extended to others teachers club from different district i.e. Bantul district of Yogyakarta. In this district, lesson studies activities were carried out in 3 junior high schools and 3 senior high schools. In each lesson studies activities, there are also 5-6 teachers in collaborations with university lectures and Japanese experts to carry out the steps of activities. There are 4 cycles of activities in the 3rd phase. In the 3rd phase we involved more intensively the teachers club.

Results and Discussion

Results of lesson study implementation can be summarized from the activity reports of piloting program were presented in the following table.

Table: The Condition of Student, Teacher, and Supporting Teaching Aids

Aspect	Before Piloting Activity	After Piloting Activity
Student	o Low learning motivation, mathematics and physics were seen as difficult subjects o Passive participation / involvement o Low ability in using laboratory kits o Low ability in organizing data o Not skillful in making conclusion o Low ability on giving question	• Improved learning spirit and happy during the learning process • Active participation / involvement during the teaching learning process • skillful in using laboratoty kits • able to organize data dan making conclusion • able to give question and argument during dicsussion • able to cooperate with friends in

	and argument o low cooperation in group	group
Teacher	o High domination during the teaching learning process o Speech-based Instruction o Low collaboration with other teachers in teaching activities o Low of preparation of teaching material	• Low domination during teaching learning process • Student-based instruction • High colaboration with other teachers in teaching activities • High preparation of teaching material
Supporting Media	o Low in using teaching media/aids	• Good in using teaching media/aids

The results are further stated that there were indications that in lesson studies activities :

1. Students were good on learning motivation, skill-process, knowledge, enthusiasm of doing cooperation, and good communication.
2. High motivation of teacher to follow teaching process since preparation, implementation, and reflection.
3. Most MGMP teachers made good preparation (planning) and teaching performance (implementation) in front of students, university students as well as lecturers.
4. Improved student' role on learning, good teacher's role, available hands-on activity, available minds-on experience reflecting three main characteristics of ideal Scients and Mathematics (MIPA) teachings such as: hands-on activity, group work, and discussion.
5. Teachers accepted any suggestion and critic upon their teaching activity.
6. Headmaster supported much the implementation of lesson study.
7. There was a complete teaching set in each piloting class.
8. There was positive role of leturers toward lesson study as facilitator, motivator for all participants since planning, implementing, and reflecting followed by good understanding about school, collaboration with teachers, and feedback data for their lecturing.

Some problems found in piloting activity was teachers need to work longer and harder to make preparation collaborated with other piloting team. The question, then was how to make the activity become teacher's habit so they will be happily in running piloting activities.

Other problems for schedulling lesson studies activities team were:

1. there were many indifferent schedule among pilot schools causing some activities posthoned or canceled;

2. all member of piloting team were bussy people who difficult to attend all piloting activities on time.
3. How to make good communication and coordination among piloting team and school as well as among teachers would be the key to find good solution for those problems.

Some constrains and problems, however, were found during the implementation, that are:

1. the development of a good system and good communication among schools, and between schools and LPTK in conducting lesson study.
2. the support of policy and finance from goverment, both national and local, or other sponsors.
3. commitment from teacher, especially headmaster as the key word of conducting lesson study.

Recommendation

In training, teachers learn about how to do lesson study while lesson study already implemented was collective work of groups of MGMP teachers, university students, and lecturers. In making lesson plan, it was done collaboratively among them, implemented by one chosen teacher, and evaluated together through reflection. Lesson study means learning a learning activity. Teacher can learn about how to do learning activity through teaching activity (live/real or video). Teacher can adopt the method, technique or teaching strategy, teaching media used by teacher in order to be imitated and implemented by other teachers in their own classes. Other teachers or observers need to make analysis or evaluation toward the teaching activity from minute to minute. The analysis result is important as input for teacher, presenter, to improve his/her teaching while for observers, they can learn about the innovation on teaching done by other teacher.

Considering deeply on the meaning of lesson study activity, it is important to develop it among MGMP teachers. Teacher or school can open their innovative class to other teachers. In future, lesson study is expected to be one model of teacher's training with good planning, by inviting a number of teachers to attend an innovative class. It, therefore, need to improve teacher's competence as teaching agent in creating innovatice teaching activity based on students' characteristics and the demand of the progress of science and technology.

The thoroughly activities of lesson studies lead to formulate the following recommendations:

1. Lesson Study is in the line with teachers motivation to improve their quality of teaching. It need to introduce more effectively in order that the teachers need to implement Lesson Study.

2. Lesson study, with its preparation, implementation, and reflections activities, encourage the teachers to improve their teaching method; therefore it needs to improve those steps.
3. The policy of education decentralization which place teacher as the central key having widely responsibility becomes a vital aspect in developing the teaching conducted. Therefore, it needs to consider lesson study as the way to improve mathematics teaching quality.
4. The existence of MGMP in each regency has its strategic role to socialize lesson studies activities and its results.
5. The heterogen quality of teachers seen form the aspect of commitment, motivation as a teacher and competence enables the improvement its quality from teacher to teacher which automatically improving teacher collegiality in strugling their need to improve the teaching quality.
6. Lesson Study is able to be carried out in each Institute Teacher Training
7. Lesson Study can be a model for teachers' professional development.

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LESSON STUDY TO DEVELOPMENT APPROACH OF PROBLEM SOLVING
- THROUGH THE CLASS OF ADDING AND SUBTRACTING OF FRACTION-

Kazuyoshi Okubo and Hiroko Tsuji
Hokkaido University of Education, Japan

1. Introduction

We established the Practice Study Group of Arithmetic Education about 15 years ago in order to aim at improvement of a class and to raise each other's abilities focusing on the young teachers in Sapporo city.

Although there are the Council of Sapporo Educational Research as a public research organization and the Hokkaido Society of Arithmetical and Mathematical Education as a private research organization as an organization which studies an arithmetical classes, they are in the environment where many teachers having high ability take part in the meeting, therefore it is difficult for young teacher to speak for one's thought freely at the meeting.
Then, there was a proposal of wanting to make the study group which can speak one's thought freely from a young teacher, and we took the lead and established the Practice Study Group Arithmetic Education.

At this group, we have the meeting about once in a month, and about four open classes per year by a member's teacher are performed.
Teachers of our group have visited the school and have observed each other class, and have spoken freely about the classes in order to raise the teacher's ability.
For this class participation, we submit a request document of class visit to the principal of the school where a member has belonged to and then it is not that we have not been able to take permission so far.

In this paper, in order to raise teacher's ability based on concrete practice example (Adding and subtracting of fraction in the sixth grade in elementary school) about the study method, it tackles and introduces about what kind of practices and "good practice" for Teaching and Learning Mathematics through Lesson Study provided in our meeting for the studies.

2. Mathematics Education for Enhancing Student's Creativity
: Through Instruction by Problem Solving Methods

Classroom instruction based on problem solving is a method of instruction that makes the most of activities by children based on their own initiative and judgment and puts an emphasis on the process of the children themselves finding solutions to problems.

Instruction in mathematics tends to center on acquisition of knowledge and skills on the basis of explanations by the teacher and repeated drills. With "instruction centering on teacher-led explanations" and "instruction centering on drills" it is hard to get children more interested in and enthusiastic about mathematics and to feel that it is really interesting. On the other hand, "instruction based on problem solving" is instruction that aims not only at developing an "ability to think" and "ability to solve problems" but also at cultivating things like an "active attitude toward classroom learning" and "ability to make active use of mathematics" and at getting children to experience how much fun thinking can be and through that nurturing interest in and enthusiasm for and an active attitude toward mathematics. Let us consider below why it is necessary to make such an improvement of shifting to instruction based on problem solving.

2.1. What is Coming to Be Expected of Mathematics Teaching in Japan

A. Certain Root of Rudiments and Basics

It is considered that efforts toward helping children acquire the rudiments and basics of mathematics should be integrated with the aim of getting them to think on their own and express their own character and individuality. Furthermore, it is considered that acquiring rudiments and basics means not only knowledge and skills but also includes abilities and attitude in learning content the core of which requires thinking in mathematical terms and in problem solving. It is considered necessary, for the sake of being able to carry out instruction intentionally based primarily on guidance for learning of rudiments and basics, to get as clear a grasp as possible of the content.

That being the case, let us divide rudiments and basics into two general aspects and explain each of them.

(1) The Content Aspect

One aspect of rudiments and basics can be considered to be the content. It includes the contents in the textbooks, in other words the content generally considered as knowledge and skills that is divided into the instructional content for each grade. That means content known as things like "addition up to 10," "multiplication up to 9 times 9, "calculation of fractions" and "measurement of angles" and the knowledge, understanding and expression and processing skills acquired through them.

Another part of the content aspect is thinking in mathematical terms as the basis for producing knowlege and skills. It can be considered to form the core of the content of rudiments and basics. It is necessary to foster an ability to become aware of content, appreciate its usefulness and learn to apply it to other things on the basis of the child's development up to the present grade in school through the content of the instruction in the different areas of mathematics. The following are examples of thinking in mathematical terms:

- Thinking in terms of units, expressing numbers in terms of "place", thinking in terms of rate, ratios and thinking in terms of functions
- Thinking logically—drawing analogies, reasoning inductively and reasoning deductively
- Thinking in terms of functions and paying attention to constituent elements in figures

(2) The Method Aspect

The method aspect of rudiments and basics can be considered to consist of things such as the problem solving ability and learning ability. Although not all of the method aspect can be distinctly distinguished from the content aspect, it is considered a good idea to distinguish the following kinds of abilities and attitudes in instructional practice:

- Proceeding with classroom instruction on the basis of the children's own questions concerning what is being sought and how to find it
- Letting the children themselves form a general idea on how to solve the problem, themselves plan how to go about it and themselves find the answer on their own
- Encouraging the children to utilize already acquired content and experience and develop it further
- Having the children take notes on the classroom proceedings to be used in group exchanges and self-evaluation
- Encouraging them to actively communicate with one another so as to learn from one another as a group

Interest, enthusiasm and attitude are important in terms of stimulating intellectual curiosity, thus serving as a driving force in getting children to willingly and actively come to grips with mathematics as an object of learning. They are considered as mental tendency regarding the different viewpoints of thinking in mathematical terms—expression, processing, knowledge and understanding and way of learning based on problem solving ability—and as a support for the content and method aspects of rudiments and basics.

B. Emphasis on Children's Own Initiative

There should be more emphasis placed on children's own initiative in classroom learning of mathematics. It is important that children discover the meaning of quantities and figures and come to have an awareness of mathematics and increase the depth thereof through experience such as observation and experimentation and moving their bodies inside and outside the classroom.

The different ways individual children think should be given importance in instruction of mathematics. Furthermore, by exchanging their ways of thinking, children are able to acquire more versatile viewpoints. In classroom instruction deductive, inductive and analogical reasoning are frequently required of children. Also, in many cases they can solve new problems using knowledge and reasoning that they have already learned. In that sense what is being asserted here can be expected to very much contribute to nurturing the basis for their creativity.

It is also important to nurture in children the attitude of making active use in their everyday lives of what they learn about mathematics in the classroom. For that purpose, it is essential in teaching mathematics to relate it to everyday phenomena and to help children understand that everyday life contains lots of mathematical problems. One significant way of so doing is to encourage them to pose problems of their own using what they have learned in mathematics class. For instance, after they have learned the meaning of "2 + 3" and how to calculate it in mathematics class in the first grade, the teacher can ask them to formulate problems concerning situations in which the answer can be obtained in terms of "2＋3".

In order to attain that goal it is also important to provide them with such training that makes it possible for them to express themselves in mathematical terms in everyday situations.

C. Emphasis on Enjoying Mathematics

Mathematics should be taught in such a way that children enjoy it and obtain satisfaction from it. The basis for making mathematics fun for children is to help them feel that they understand it, which will lead to the feeling that "thinking mathematically is fun." That being the case, the teacher has to show ingenuity in mathematics class from the viewpoint of showing how much fun and how interesting and worthwhile it is to learn mathematics and how wondrous it can be. If the children use the mathematics that they have learned to solve problems in various situations around them, they will learn to appreciate how much fun and how useful it is learning it.

It is also important to teach children through mathematical activities how much fun learning mathematics is. There should be many situations in mathematics in which children can experience a sense of discovery and even excitement and express it in words like "Of course!" and "Yeah, I see!" For that, children have to be encouraged to think for themselves. Just listening to the teacher's explanation and doing a lot of drills will not result in the feeling on the part of the children that mathematics is interesting and even fun because they will often end up thinking that they "can't do it" or "don't understand" when they run up against more difficult problems.

There ought to be a lot of situations in mathematics class where children can encounter discovery, emotion and satisfaction of attainment. What it takes to make mathematics seem interesting and fun is to have them experience those feelings as often as possible.

The more children come to like and enjoy mathematics through experiencing how interesting and even how much fun it can be, the better. We must not give up on children who have not been very good at mathematics so far. They, too, can learn to think "That mathematics class was interesting." We must not continue with teaching methods that produce feelings in children like "I don't want to do mathematics anymore!" and "Thank goodness there isn't mathematics anymore!" What we have to aim for is the kind of classroom instruction that can turn the consciousness of children concerning mathematics in the direction of "mathematics is really interesting!"

What is required of school education is that it accomplishes firm rooting of rudiments and basics in children's minds and turn out children who are able to learn themselves, think on their own, use what they have learned and show creativity inside and outside the classroom. Furthermore, the aim of the kind of instruction of mathematics in the classroom described above is acquisition not just of knowledge and skills but also of capabilities and positive attitude regarding mathematical thinking, learning focused on problem solving, and so on. When engaging in instruction that intentionally puts the accent on acquiring rudiments and basics, it is necessary to have an attitude of instruction characterized by effort to grasp the content of the instruction as clearly as possible.

It is important that the children understand the meaning of and comprehend mathematics and that they develop the ability to apply the content and methods taught in mathematics class in order to solve problems that arise in their everyday lives. That goal cannot be attained with instruction in only one direction and with teaching that results in acquisition of what seems like knowledge and skills but really is not. If attention is paid to the children's process of thinking and if they share their thinking with each other, they will be able to see things better and think better, and that tendency will spread. That is why "instruction based on problem solving" is considered to be the most appropriate method of teaching mathematics.

3. Concretization of Instruction by Problem Solving Methods

3.1 Procedure of Instruction Based on Problem Solving

In order to build instruction based on problem solving it is necessary to consider what makes instruction characterized by emphasis on acquisition of rudiments and basics, and that entails inclusion of the viewpoints of, for example, setting of a clear image of such instruction, sorting out the problems that have to be ironed out as regards instruction the way it has been up to now, integrating such problems with improvement through shifting of the accent to problem solving and rethinking evaluation of learning. In that connection it is important to consider the following points:

- Awareness of the overall curriculum plans for mathematics
- Formulation of concrete instruction plans for the different units of instruction

- Definition of how the class hour of instruction is to proceed and how the situation regarding acquisition of rudiments and basics is to be determined

The following points are also important in the case of instruction based on problem solving:

(1) Achieving the result of initiative on the part of the children themselves in instruction based on problem solving.

(2) Preparation of materials for the instruction that are as far as possible suitable for the content to be taught and in tune with the needs and lives of the children.

(3) Setting of instruction goals in tune with the actual conditions of the children and the Educational tasks of the school and relating the difficulties of acquisition of rudiments and basics with the methods of evaluation of such instruction.

(4) Supporting activities that stimulate the enthusiasm and problem awareness of the individual children and that encourage them to think and pursue solution on their own.

(5) For group discussion activity that can lead to better problem solving, changing from the kind tailored to the teacher to the kind based on the viewpoint of the children themselves that can serve as a forum for discussion and communication in which they themselves share their values.

In general, instruction processes such as those indicated below (underlined) come to mind as regards classroom instruction based on problem solving, the aim in each process (step) being acquisition of ability and the necessary attitude concerning problem solving.

- <u>Formulation of the problem</u>
- <u>Understanding of the problem</u>
- <u>Planning solution of the problem</u>
- <u>Carrying out the solution</u>
- <u>Consideration of the solution</u>

Here, in accordance with such ideas, there is setting of instruction processes A to E as partial revision of some of them, with the goal of attaining that aim.

A. Understanding and grasping the meaning of the problem (collecting and sorting out information constituting the problem and formulating the problem oneself, getting familiar with the problem situation regarding the given problem and conceiving it as one's own problem)

B. Planning solution of the problem (preparing the conditions and information needed for solution (already acquired experience and knowledge, skills, ways of thinking, etc.) and getting a rough idea about how to go about finding the solution)

C. Carrying out problem solving (reaching a tentative conclusion concerning the content of the solution (formation of concepts, acquisition of knowledge and skills, becoming aware of mathematical ways of thinking, etc.) through trial and error in mathematical activities)

D. Consideration of the solution (collating and checking the results with what was expected, the children's considering the different contents of each other's solutions as a group and arriving at a more refined solution)

E. Final summing up and looking back on the processes of solving the problem (confirmation of the state of attainment of the goal (things like state of acquisition of rudiments and basics and realization of the evaluation criteria) as a basis for own evaluation of classroom activities)

Classroom instruction based on problem solving is a method of instruction that emphasizes the children's activity based on their own judgment and the process of solution by the children themselves in working toward the goal of the instruction. It is therefore important that the teacher present problems suited to that goal and work to support the children's own independent activity. Particularly important are the questions that the teacher throws out during the class. That being the case, in the guidebook the main problem to be dealt with in the class period is put in a rectangular box, and the questions thrown out by the teacher are written in gothic script.

3.2 The Significance of Question Posing in Classroom Instruction Based on Problem Solving

In the course of the process of classroom instruction the teacher is expected to talk to the children in such a way as to strive to get them to better manifest their thinking and behavior, exploring their individual inner minds and understanding their individual characters and personalities. Let us define such "putting questions to and spurring" the children individually and as a group by the teacher in agreement with such a desirable picture of classroom instruction based on problem solving as "question posing."

What question posing should be like needs to be ascertained in an integrated manner with the task of improving classroom instruction in the way problem solving is presently done as indicated above? In other words, ingenuity and improvement in question posing should not be just for the smooth progress of instruction by the teacher but rather should be for the purpose of achieving improvement in inadequate points and points in which sufficient results are not obtained in as-is classroom instruction, focusing on "how to get the children to make progress" by proceeding with classroom instruction on the basis of problem solving.

Furthermore, question posing should not be a one-sided affair, but rather aimed at getting the children to react and respond; the point of ingenuity here is to lead to "dialogue" both between the teacher and the children and among the children themselves, which is essential to establish communication in the classroom based on that.

Question posing is considered to be the function of eliciting and assisting the children's thoughts in connection with acquisition of knowledge and skills and formation of mathematical way of thinking and necessary attitude. In eliciting the thoughts of individual children, one should not expect them to be appropriate and valid as a complete whole, and the direct purpose should not be that of having them announced to the whole class as such, but rather the basic aim should be that of simulating the children's inner minds and thought processes.

Response is elicited by stimulus. But priority should not be given to getting response for the sake of convenience of the teacher in his or her instruction. Rather, the main point should be using response for promotion of the child's thinking activity and getting the children to talk with one another about their thoughts for deeper appreciation of them among them.

3.3 Example of Question Posing in Classroom Instruction Based on Problem Solving

Teaching Plan of a Mathematics Lesson

Students: Sixth grade, Elementary School in Sapporo
20 boys and 17 girls, total 37 pupils
Teacher: Masu Kanno

1. Unit : Adding and subtracting Fractions

2. Aims and the flow of learning fractions

(Aims)

- Interests, attitudes, motivation

To understand the situation where adding and subtracting fractions with unlike denominators are used and willingly try to solve the problem with the knowledge already acquired.

- Mathematical thinking

To understand it is possible to solve the problem by using diagrams or making the denominator, which is the measuring unit of quantity, the same number. Then, to think up reducing fractions to a common denominator as the way of calculation.

- Expression, skill

To be able to simplify fractions and to be able to convert the fractions to have the same denominator.
To be able to calculate adding and subtracting problems of fractions.

- Understanding, knowledge

To understand the meaning and the method of simplifying fractions and converting fractions with unlike denominators to fractions with a like denominator .

(The flow of learning fractions)

Fourth grade	・The meaning and the notational system of fractions
Fifth grade	・Adding and subtracting fractions with same denominators ・Equivalent fractions—simple case ・Writing the answers of dividing whole numbers in fractions ・Relating fractions to decimals, relating decimals to fractions
Sixth grade	・Equivalent fractions, how to make equivalent fractions ・The meaning of simplifying fractions and reducing fractions to a common denominators ・Adding and subtracting fractions with different denominators ・The meaning and calculation of multiplying fractions ・The meaning and calculation of dividing fractions

3. About This Teaching Material

(1) The value of this teaching material

The main aim of sixth grade's "Adding and subtracting fractions" is to deepen the student's understanding about the meaning of fractions and to develop their ability to calculate fractions.

The concrete teaching items are

a. to understand that the fractions made by multiplying the same number to the numerator and the denominator has the same amount.
b. to put together how to check equivalent fractions and how to compare fractions
c. To be able to calculate adding and subtracting fractions with different denominators.

In the fourth grade's "fraction", the students have learned the meaning and the way to write fractions. In some simple cases they have learned there are equivalent fractions. In addition to that, they have learned adding and subtracting fractions with the same denominator and expanded their view of numbers and calculations.

Also, they have studied adding, subtracting, multiplying and dividing whole numbers and decimals. Whole numbers and decimals are written in the decimal system and they have deepened their understanding and skills of the four rules of arithmetic within the system.

In this teaching unit, they study adding and subtracting fractions, which is not written in the decimal system. Accordingly, we have to help them understand the meaning of fractions through many different situations. In order to do that, at the introduction of this unit, I let them make questions to link the meaning of a concrete situation and the adding and subtracting fractions. Also, by using these questions throughout the unit, they will be able to have the perspective of the whole unit. Furthermore, as the study goes with their questions, we can expect their enthusiastic attitudes.

The meanings of "reducing fractions to common denominators", "simplifying fractions" and the method of adding and subtracting fractions are tend to be taught in a mechanical way. However, we would like to make the most of student's ideas and organize a lesson as if they find things by themselves and feel the merit of using the idea of common multiples in fractional calculations.

(2) The ability we want to cultivate in a student

(With reference to the content)

a. Adding and subtracting fractions become possible by reducing the fractions to a common denominator.
b. The adding of mixed fractions is the adding of "whole number + proper fraction"s based on the idea of a measuring unit.

c. To understand that there are many ways to write the same equivalent fraction by using diagrams.

(With reference to the aims)

a. Mathematical thinking

To help them find the rules to make equivalent fractions.

$$\frac{a}{b}=\frac{a\times c}{b\times c}(c\neq 0),\frac{a}{b}=\frac{a\div c}{b\div c}(c\neq 0)$$

b. Logical thinking

There are many equivalent fractions. Simplifying a fraction means to express it in the simplest form, i.e., to express it by using the smallest denominator. Reducing fractions to a common denominator means to express each fraction by using the same denominator.

In adding and subtracting fractions, it is important to express the fractions by using the same denominator (that is, the unit of measuring) and think in an orderly fashion.

c. Generalization

Through the learning of adding and subtracting fractions with unlike denominators, the students learn the points in common and the points of difference in calculating whole numbers and decimals. They pay attention to correlations among the groups of numbers.

d. Estimation

i) The prospect of a unit:

In this unit, making problems by students are taken in the first class of mathematics and understanding of the phenomenon in which there is a case using adding and subtracting with different denominators fractions is aimed at, and formula representing of the phenomenon is performed. The whole unit is constituted using these problems made by students, thereby, it is seemed that children can foresee the contents of study of the whole unit.

ii) The insight of reduction of fractions to a common denominator:

In this lesson, we take the subject of (unit fraction)-(unit fraction), and want children to discover the necessity of changing fractions to common denominator or reduction of a fraction.

It seems that it is easy to find the fraction as a unit for students by themselves if it is introduced from the scene of subtraction rather than the addition scene. Moreover, I want to make estimate a solution at the time of introduction. This activity will also help for students to finds the fraction used as a unit.

In including these two classes, the next two classes the study of reduction of a fraction and changing of fractions to a common denominator will be carrying out, then we will take 4 classes of various different denominator fractions. Student will study these classes by themselves since we consider it is possible if students got how to study the adding and subtracting of different denominators unit fractions.

iii) Estimation of solution:

Also here, students will estimate a solution, paying attention to [merit of changing fractions to a common denominator] = [the merit of a common multiple], it seems that students can realize the merit of estimation in the conclusion of 10 classes studies by themselves.

iv) The prospect of domain:

For studying the domain of "numbers and calculations", usually the following order is taken: "Understanding of a phenomenon" ➡ "formula representation" ➡ "study of algorithm" ➡ "application". Thus, if the order of progressing study is known, when students advance study, it will be effective.

It expects to make it food for the following study of the domain of "numbers and calculations", when students understand the merit of learning in such an order from the studies in this unit.

For the reason, it is thought to perform the looking back the whole unit in the end of a unit.

4. Teaching Plan (13 hours)

1st	Let's make problems of adding and subtracting fractions.
2nd /3rd	(See 5.)
4th	Is it possible to subtract fractions if the denominators are the same? It is possible to subtract fractions with like denominators. To find common denominator, it is easy and fast if we use common multiples.
5th	Do the fractions $\frac{1}{6}, \frac{2}{12}, \frac{3}{18} \ldots$ have different values? The values of fractions are equal if both the denominator and numerator of a fraction are multiplied or divided by the same number.
6~9th	1. Adding proper fractions (No carry up) 2. Adding proper fractions (Carry up, simplify fractions) 3. Adding mixed fractions (Carry up, simplify fractions) 4. Subtracting proper fractions from mixed fractions (Carry down) 5. Subtracting mixed fractions from mixed fractions (Carry down) 6. Addition and subtraction of three fractioins
10th	Discussion

11th	Let's express time in fraction
12~13th	Practice: Encourage each students to learn

5. Detailed Plan of the Second and Third Classes (Appendix I)

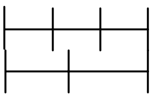
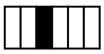
Aim:

To find the common measuring unit during the activity of comparing quantities.
To notice that subtracting fractions with unlike denominators is possible if the fractions are expressed by using the same denominator.
To express quantity by using diagrams or equivalent fractions. To try to calculate the difference between them.

6. The device of question posing in this class

Question Posing	Content (corresponding thinking)
1) (After getting a formula 1/2 -1/3) How can you estimate the solution? (Nice question: Is an answer reach to 1?)	From the information acquired from the problem sentence or formula, students estimate a quantity of the answer (difference) in question.
2) Which part of the area figure is asked in this problem?	In the area figure which the child showed, it clarifies which portion hits the answer (difference) of this problem. (The clarification in question)
3) Why can J-kun understand this part is 1/6?	The basis and reason of the idea are clarified for whether how did it consider and found the difference of one half and 1/3 was 1/6 (Reason and deduction)
4) How did K-kun consider it was how many parts of the remaining part?	When the difference of 1/2 and 1/3 is planning how many parts of the remaining to have paid their attention to being 1/6 when compare it with 1, find a common unit, and clear it to an equivalent fraction; is planning to have particularly paid its attention, and a classification arranges it. (Clarification of a thought)
5) Today, we learned subtraction of fractions. How could you calculate it?	Promoting the rearranging of learning of today's class, students notice to be able to calculate a subtraction of fractions if they make same denominator of two fractions. (Generalization)

Appendix I

Learning Activity	Student's thoughts	Teacher's Activity
Which of $\frac{1}{2}l$ milk and $\frac{1}{3}l$ juice contains more and how much is the difference?	Probably the answer is smaller than 1/2.	Let the students imagine and understand the situation of the problem.
↓ Expression: $\frac{1}{2}-\frac{1}{3}$	I can't subtracting because the denominators are different.	
a. $\frac{1}{2}-\frac{1}{3}=\frac{1-1}{2-3}$ b. compare the difference of 1/2 and 1/3 with the whole (Numerical line) (Area diagram)  	If they have the same denominators, it's possible… I'll compare them by drawing diagrams. I know which contains more, but I don't know the difference. Let's compare the difference with the whole.	Encourage them to estimate the difference. Encourage the students who can't have the perspective to think in the diagram or remember what they had learned before.
(Tape diagram) 	The amount of 1/2 and 1/3 are 3/6 and 2/6 if the nicks are changed. Then the difference is 1/6.	Teach each student while walking around the classroom.
c. Compare the difference by finding the common unit. (Tape diagram) (Area diagram)	We had learned that 1/2=3/6, 1/3=2/6 in the fourth grade. That means the difference is 1/6. It is possible to subtracting if the denominators are the same.	Let the students pay attention to the relation of the diagrams and the numerical expression.
(Common multiples)	Is it possible to subtracting fractions in other cases?	Let them consider in other problems.
It is possible to subtracting fractions with unlike denominators if we write the fractions by using the same denominators. Is it possible to calculate other subtracting problems of fractions? How can we find the fractions with same value? We got many answers. Do they have different values or not?	There are many fractions which have the same value. How can we find them?	Let them consider what they want to learn next.

A STUDY OF “GOOD” MATHEMATICS TEACHING IN JAPAN

Takeshi Miyakawa

CRICED – University of Tsukuba, JAPAN

The aim of this paper is to advance understanding on the characteristics of the lesson which is often recognised as a “good” lesson in Japan from two perspectives: learning process and teaching process. A case study will be carried out using the theory of didactical situations on a videotaped lesson which according to Japanese standards is a “good” one.

INTRODUCTION

What is “good practice” or a “good lesson”? The adjective “good” is subjective. We do not have an absolute criterion for “good”. Also, what is “practice”? This term also indicates different activities. When the adjective “good” is used, the object has to at first be clarified. So that the participants may share the ideas on mathematics teaching activities of different countries, on the occasion of the previous APEC specialist session in Tokyo, a discussion was raised on these questions. It seems that the meaning of the later term, “practice”, has been well clarified among the participants: it refers to the teaching practices in a classroom on the one hand and on the other the teachers’ practices which allow their professional development and consequently the improvement of teaching practices in the classroom. The “Lesson Study”[1] well developed in Japan is thus often recognised as a “good practice” in this later sense[2].

However, the answer for the first question about “good” was not easy to discover. In the case of “good” teaching practices or lesson, some criteria based on different viewpoints were proposed, out of which here below three of them are summarised.

- Teaching process
  One way to define “good” is by the teaching method. A “good” lesson is given by adopting in the classroom a method recognised by the teachers as a “good” approach for teaching. For example, the lesson by the open-ended approach (cf. Becker & Shimada, 1997) is often recognised as a “good” lesson in Japan.
- Learning process
  The postulate of the constructivists, “Pupils construct their own knowledge, their own meaning” (Balacheff, 1990, p.258), supposes that if the learning in the classroom were conducted so that the pupils or students could construct knowledge and meaning by themselves, the lesson would be successfully carried out and the teaching practice used would be a “good” one. To evaluate a given class, in this case, students’ learning process should be precisely analysed from the constructivist point of view.
- Students’ achievement or outcome
  The other way to define “good” is by assessing students’ results. The lesson is recognised as a “good” lesson, if students have achieved well in the mathematics assessment. For example, we may evaluate students’ progress from the results of the national or regional assessment. The students’ achievement should be rated by the goals of the curriculum or the lesson. Some goals can be assessed by a simple paper

test, but others cannot be. The later is usually recognisable in the learning process. Therefore, this criterion of "good" overlaps the second criterion.

These criteria are often used together to identify or discuss on the subject of a "good" practice or lesson preparation.

The aim of this paper is to advance understanding on the characteristics of the lesson which are often recognised in a "good" lesson in Japan from the first two points of view stated above: learning process and teaching process. A case study will be carried out on a videotaped lesson which according to Japanese standards is a "good" one.

Theoretical Framework

The global image of teaching in Japan compared with that of Germany and that of United States has been enunciated in TIMSS video studies: "In Japan, teachers appear to take a less active role, allowing their students to invent their own procedures for solving problems" (Stigler & Hiebert, 1999, p. 27). The motto for Japanese teaching has been called: "structured problem solving" (ibid., p. 27) while the Japanese lesson pattern has been characterized through comparison with patterns of other countries by a sequence of five activities (ibid., pp.79-80)[3]:

- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major points

These patterns describe the overall activities which are conducted in the classroom. In order to analyse more precisely the characteristics of "good" teaching practice in this paper, the theory of didactical situations (Brousseau, 1997) is adopted as a tool of analysis. It is not a teaching method nor an evaluation method of the teaching practice, but provides us with a model for the analysis of an effective classroom in order to understand what processes are taking place in terms of students' learning. At the same time this theory allows us to identify the relevant learning and teaching situations (didactical situations) with reference to the mathematical situations.

In this theory, the Piagetian postulate for the learning is adopted: "The student learns by adapting herself to a milieu which generates contradictions, difficulties and disequilibria" (ibid. p.30). In order to characterise different process of learning and teaching of the target mathematical knowledge, four situations – action, formulation, validation and institutionalisation – according to the stages of lesson are taken into account. And the notion of "devolution" is also an important element for the analysis; it's a process in which the teacher puts the student in an "adidactical situation" (ibid. p.30) where the student solves the problem on his own responsibility. The learning and teaching situations are modelled by the notion of "games". The student's games are to play "with the adidactical milieu which allow the specification of what the function of the knowledge is after and during the learning" (ibid. p.56), and the teacher's games are to organise student's games. A didactical situation is therefore expressed as the diagram (Figure 1).

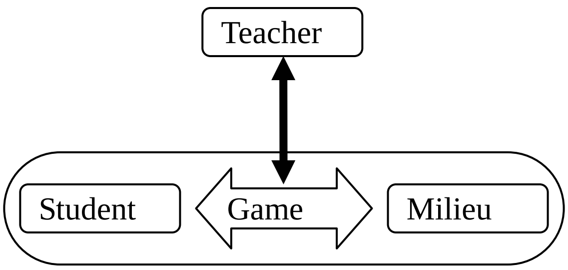


Figure 1: cf. Brousseau (1997, p.56)

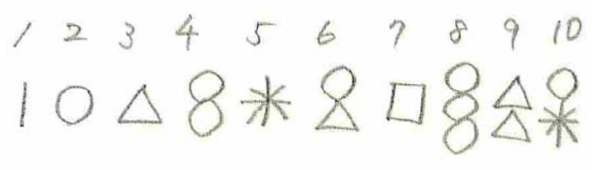
Theoretical Analysis of the Tasks

The lesson I selected is one which is given by a teacher of the elementary school attached to the University of Tsukuba. This was a part of a “lesson study” that is demonstrated on the occasion of APEC-Tsukuba conference in January 2006 in Tokyo. The teacher of attached school is recognised as a practised expert teacher.

Lesson topic: prime and composite numbers

The lesson plan written by the teacher is attached to the appendix of this paper. The target mathematical knowledge is the prime and composite numbers. The goal of the lesson is for pupils to be able *to view a number as a product of other numbers* (see the appendix). It will of course include the understanding of the fact that some numbers cannot be a product of other numbers except the identity element “1” and themselves. For example, the number 12 can be seen as a product of the numbers “3” and “4”. The lesson which will be analysed in this paper is the first of two consecutive lessons. The first one is the introductory lesson. We can find from the lesson plan the two main tasks proposed in the lesson:

Task 1: The cards are ordered. Identify the implicit “rules”.



Task 2: Using the discovered “rules”, how 11 and 12 can be expressed?

Each card has just symbols. The numbers in the above diagram are not given. The teacher’s expectation of task’s result is that pupils find the rules which allow them to accomplish the second task. Therefore they have to find and recognise that the circle corresponds to the number 2, the triangle to the number 3, the star to 5, etc. and some symbols together have a relationship of multiplication. If these activities are considered “games”, like in the theory of didactical situations, there are two games: the finding of implicit rules and the finding of the symbols for 11 and 12.

From the viewpoint of representation of the number, numerical representation and graphical or pictorial representation are taken into account. In the activities, alternation between numerical representation and graphical representation is proposed. The numbers expressed by numerical representation are not shown on the cards, but revealed in the teaching process. The advantage of the graphical representation is that it shows visually the structure of the number and the number system in terms of prime numbers, in other words, the composition by prime numbers. This point is very often concealed by the numerical representation.

Analysis of Task 1: implicit rules

For task 1, as the task for pupils is just to find implicit conventions, we may consider several rules. Some of them are operational for the second task and some of them are not. By clarifying the rules supposed to be found by the pupils, this analysis will help us grasp the nature of rules the teacher expected to be found by the pupils in an effective lesson. While analysing the rules, I made the distinction considering the nature of the statement, especially the validity of the statement, between the "descriptive rule" and the "hypothetical rule".

The former is a descriptive statement which is true in a given order of cards. It can be operational when it is applied to the cards whose symbols are unknown. For example, the following are descriptive rules which can be considered a priori, however their list is not exhaustive:

1. Some cards have only one symbol, whereas others have plural symbols.
2. If the number becomes bigger, the number of symbols increases as well, with some exceptions.
3. There is at least one circle in every two cards.
4. The even number has at least one circle.
5. There is at least one triangle in every three cards.
6. The numbers multiplied by 3 has at least one triangle.
7. The prime number has always one symbol, not more, and different from others.
8. The composite numbers have always more than two symbols.

The later rule is a general statement which is hypothetical but whose validity can be checked empirically in the given cards. For example, the followings are those ones:

9. A symbol represents a number.
10. A symbol represents a prime number.
11. A circle represents the number "2".
12. A triangle represents the number "3".
13. The composition of symbols represents the multiplication of numbers.

This distinction will be important because the descriptive rules can be found or stated directly from the observation of given cards and numbers, whereas the hypothetical rules need to be verified by using other hypothetical rules. Moreover, it's the descriptive rules that allow pupils to formulate hypothetical rules and these are the kind of rules expected by the teacher to be found.

The rules stated above are "correct" from the viewpoint of the teacher's expectation. We may also consider "false" rules. For example, the composition of symbols represents the addition of numbers. I want to also mention the relationship between the rules in the sense that there is a hierarchy among rules. In particular, in order to find and apply rule 13, some other rules, such as the rules 9, 11 and 12, should have been discovered and applied. This means that whereas the question posed for task 1 is open, some specific rules are required for task 2.

Analysis of Task 2

The second task is the "game" to find the graphical representation of given numbers and consists of two sub-tasks. The first one of these is to find the symbol for the number "11" and the second is to do the same for number "12". To accomplish the first sub-task, the discovery of the descriptive rule number 7 from the above list will make pupils anticipate the symbol on the 11^{th} card as a single symbol. For the elucidation of the second sub-task several rules should be employed. A brief analysis of the nature of this second sub-task is included in the paper for clarification purposes.

We may consider as an approach for the resolution of the second sub-task the factorisation of given numbers. The process of resolution is as follows:

1. factorise at first the given number “12” in the numerical representation (e.g. 12 = 2 x 6);
2. find the symbols which correspond to the numerical numbers obtained by the decomposition (e.g. “2” to “o”, “6” to “Δo”, etc.);
3. draw them together one below the other.

In the first step, for the factorisation, division and multiplication are available. In the case of multiplication, pupil will find heuristically two or three numbers, multiply them, and verify whether their multiplications will be “12”. The pupils who cannot decompose are not able to reach the answer. What indicates to the pupils they should factorise the given number are primarily the rules 8 and 13. In fact, if the composition of symbols is not recognised as number multiplication, the factorisation cannot be done. Hence, this step requires to implicitly or explicitly use rule 13 to accomplish the task. The second step consists of discovering the correspondence between the numerical numbers and the symbols. The hypothetical rules 11 and 12 are required. The third step will be solved by using again rule 13.

The hypothetical rules are required to accomplish the second sub-task. The descriptive rules are not enough, and such this latter set of rules allows pupils to anticipate the symbols on the 12th card, but not the complete suite. That is, rule 3 or 4 makes them anticipate that at least one circle will be on the card; rule 5 or 6 to expect at least one triangle on the card. However, these descriptive rules are not able to make them anticipate two circles.

Analysis of the Videotaped Lesson

The lesson is videotaped and analysed using the video recording and the transcript. The actual lesson was proceeding including several activities. The analysis here will be conducted by dividing the lesson into three stages: introductory activities, activities for task 1, and activities for task 2. Each part is described and at the same time analysed. This analysis is not a part of the lesson study, but is carried out on the videotaped data from a researcher’s and not a teacher’s point of view in order to clarify the characteristics of a “good” lesson in Japan.

Introductory activities

The pupils’ activities at this stage were limited to replying questions or fulfil requirements which are chronologically described hereafter.

1. Pupils were asked what they have noticed on the cards introduced into the lesson by sticking them randomly on the blackboard. Among them two cards have no symbol;
2. Pupils had to come up with symbols that might be on the two blank cards;
3. They had to make suggestions of ways to categorise the cards in order to make clear some implicit rules (not necessary the ones they were supposed to find).

In the lesson, as there are not many criteria on which the answer to the second question can be based on, when a pupil proposes a grouping, the teacher asks whether there are or not any criteria for the classification of the cards. This question implicitly makes the pupils group the cards. The categorisation methods proposed by the pupils are generally summarised in the following two methods:

- By the number of symbols on each card
- By the combination of different symbols on each card

At this stage, we see that pupils are familiar with symbols and recognise or find the implicit descriptive rule of the distinction between one symbol and composed symbols. It also seems that the pupils are aware of the differences and similarities of the type of symbols in the given cards and symbols (see the following dialog).

22. S: The card with a bar should go to the second group because it has only one symbol.
23. T: So there are groups of one symbol, two symbols and three symbols. *This is a pattern easy to see for everyone!*
24. S: All the cards of the first group *have* a circle, so the card with two triangles and no circle should not be there.
25. T: *So you are saying* that these are all circles so the triangle should go to the other group. This can be one way of thinking.
26. S: *The cards with two different symbols belong to the first group, but the card with two circles should go the bottom group where there are cards with only the same kind of symbols.*

From the viewpoint of the theory of didactical situations, this stage is the first step of a devolution process which allows pupils to better understand the rule of the "game" that the teacher will propose later (find implicit rules in task 1 and find symbols for given numbers in task 2). No matter what, the teacher does not directly ask pupils for the next activity, but uses pupils' discourse and guides them (e.g. "so you are saying …" [25]). This way of intervention on the part of the teacher makes pupils engage in their activities out of their own responsibility. The teacher's authority does not dictate their intellectual activities. This is one of the conditions for the devolution process.

Activities for Task 1

Task 1 is proposed to the pupils [29]. The teacher sticks the cards slowly one by one on the blackboard from left to right. He implicitly indicates the order of cards. At this moment, the cards have not yet been given numerical figures. As the teacher is sticking cards slowly, the pupils anticipate which should be the next card to be put up. At this moment, the goal of the "game" for the pupils is to find implicit rules ("please tell us what kind of pattern you found" [29]) and at the same time to find the next card.

29. T: […] Now, I'm going to reorder these cards in my way. By looking at my way of ordering, *please tell us what kind of pattern you found*. The way of thinking you did will be very helpful. I'm putting the first card, the second one, the third…

Just after sticking all the cards on the blackboard, some ideas are proposed by the pupils. One of them is "S: it's a multiplication" [36]. Although this is the final rule the teacher expects to be discovered, he writes it down on the blackboard and asks to the class to find simpler descriptive rules: “T: the hint does not have to be so complicated. Can you see some more interesting rules in this pattern?” [43]. It is visible from this example that the teacher regulates the class on the path he was expecting it to go and the rules he is expecting the pupils to find at first are descriptive ones. The answers given by the pupils to the teacher’s question are as follows.

- the even numbers have circles on the card
- the triangle appears after every two cards

These are the descriptive rules. As the numerical numbers have not yet been written on the cards, the teacher asks the pupil who proposed the first descriptive rule to write down numbers in order to clarify his proposition for the other pupils. The teacher clarifies the pupil’s idea. At this moment, the rules found are not the final hypothetical ones, but the descriptive ones, which will play an important role in finding the final rule. Because of this, the teacher accentuates this rule for the pupils in the class: “T: It’s an interesting discovery! Each card positioned on an even number has a circle” [51].

Until this point, the implicit rules proposed are focused especially on the relationship between the cards and the descriptive rules have been identified. Next, one pupil mentions the relationship between the symbols as it follows, and the teacher continues from there:

56. S: The number “2” has one circle and “4” has two circles, so I thought… two and two makes four. And next is “6”…
57. T: […] The circle means two. Two and two is four. O.K? Do you understand? Two and two is four. *How else can you say that in mathematics?*

The first pupil’s proposition “two and two makes four” contains an ambiguity and also it seems that the pupil is not so certain of his answer [56]. The teacher at this moment clarifies the pupil’s discourse, and makes pupils focus on it by saying “How else can you say that in mathematics?” [57]. This question makes pupils to think about the relationship between composed symbols and formulate the idea. In the words of the theory of didactical situations, the teacher puts pupils into a situation of formulation. Until this moment, pupils act and reflect on the given task in order to find the implicit rules in the given ordered cards and symbols. Therefore, they were in the situation of action. However, the distinction between action and formulation is not obvious, because the problem is to find a formulated rule.

58. S: 2 plus 2 equals 4. 2 times 2 equals 4.
59. T: both of them are right, no? … These four people seem to say no. So, please explain why you are against.
60. S: I think it is correct that the two circles of the 4th card mean the addition of two “2”s or multiplication of two “2”s, but if so, when it comes to the 6th card, *the triangle should represent number “3”* and then we got “3” plus “2”, which is five. So, I don’t think it should be an addition.

Next, the teacher proposes the validation of given rules [59], after multiplication and addition are both proposed [58] (situation of validation). Some pupils explain that the implicit rule which they are searching is multiplication rather than addition by using the other numbers [60]. After the pupils’ explanations, the teacher summarises and verifies this rule for the other numbers on the blackboard: 2 x 2 x 2 = 8, 2 x 5 = 10. Then he states and writes down clearly on the blackboard that the implicit rule which they are searching for is “multiplication” (situation of institutionalisation).

The feedback from the milieu, when a pupil anticipates an implicit rule, will be the result of verification with other cards. For example, when the hypothetical rule “the composition of symbols represents addition” is anticipated, what validates it is the calculation result for 6 or 8 based on the other hypothetical rules. This feedback will be elicited from the milieu on condition that the pupils are aware of the rule validation method. In fact, if a pupil thinks that the rule to be found can be valid not for all but only some cards, this validation method will not be adopted. In the activities of this stage, the feedback from milieu would be situated in the situation of validation rather than in the situation of action.

Activities for Task 2

After his summarizing what has been found on the hidden rule, the teacher asks the first sub-task of task 2: what symbols will be on the 11th card? While asking the pupils, he also inquires whether the symbols which were drawn before (three triangles "ΔΔΔ") can be used for the 11th card. The answer from pupils comes out quickly: "27" expressed by three triangles is not relevant, and a new symbol is necessary for the number "11". The teacher then asks to the pupils the reason of the answer [83]. At which a pupil replies as it is expected by the teacher [84].

(pupils draw a pentagon and an X)

83. T: This is correct. This is also correct. Why do you think these are correct? Are these symbols that you drew the right ones? *Why do you think it is O.K. to use symbols like these?*
84. S: The numbers which have only one figure, take "2" or "3" for example, can be divided only by "1" or the number itself. "11" as well can only be divided by one or itself, so the card has to have only one symbol like before with the triangle or rectangle.
85. T: You are talking about the opposite of multiplication, don't you? If you think about something using a combination of multiplications, we can't express the number "11" in that way. It can be expressed by an addition, but none of these cards represent additions. Right?

The activity for the first sub-task does not take long. The essential fact that some numbers can be only divided by "1" is clearly stated by a pupil [84]. Therefore one of goals of this lesson, which is understanding of the prime numbers concept, has been reached. The new idea is repeated and clarified with examples of multiplication by the teacher [85]. The term "prime number" was not verbalised by the teacher in the lesson, due to Japan's national course of study where it is introduced in the 9th grade.

From the viewpoint of the theory of didactical situations, in order to accomplish the given task (find symbols for "11"), the target mathematical idea in the lesson (some numbers cannot be a product of other numbers except "1" and themselves) is elicited. What makes pupils to come up with this idea is the first sub-task. In particular, the teacher's question on the reason of the selected symbol formulates this idea to the pupil himself (formulation) [83]. We can also find a sign that the class is about to discover this idea appears in the remark of a pupil from the previous stage [71].

71. S: I understood it's a multiplication, but how can we come up with the number "11" by multiplication?

The second sub-task is proposed by the teacher, finding what symbols will be on the 12th card? This time, he invites them to write down on their notebook. The teacher asks a pupil and he gives a wrong answer, six circles "oooooo". This answer is corrected by others who indicate that the given symbols are wrong, at which point the pupil writes again: four triangles "ΔΔΔΔ". The problem is that he uses the unwanted rule of addition, instead of multiplication and the teacher uses this opportunity to clarify what implicit rule are they after. Before indicating that these given answers are wrong, the teacher asks a pupil to write on the blackboard answers differing from them. A girl writes two circles and a triangle (ooΔ). The teacher asks her the reason. She explains it by "4 x 3 = 12" (oo and Δ).

102. S: (the pupil draws two circles and a triangle) Because so far, as 2 times 4 is 8, there are three circles, and as 2 times 3 is 6, a circle and a triangle are combined, as 4 times 3 is 12 and 6 times 2 is 12, it will be like that (each time, she refers to the symbols of previous cards such as 2 and 4 to 8, 2 and 3 to 6, 4 and 3, and 6 and 2).
103. T: These circles make "4", and four times "3" is "12". Great! Why did you think these symbols are wrong? (pointing to the four triangles)

104. S: in the case of 8, we multiplied 2 to 2 and 2, therefore also in the case of triangle, 3 times 3 equals to 9, 9 times 3 equals to 27, 27 times 3 becomes 81.

After clarifying the girl's explanation to the class, the teacher comes back to the previous answer (ΔΔΔΔ) and asks to the whole class why it is not correct [103]. A pupil explains why [104] and the teacher clarifies her explanation. However, when the teacher asks again for the other answers, a pupil draws two hexagons on the blackboard. This pupil does not give an explanation, but the reason will be elucidated by the other pupils [112, 113]. This pupil did not use the expected rule, multiplication, but an unexpected rule, addition again.

112. S: Perhaps, as there are two symbols for "6", I thought she joins the two symbols and makes a new symbol, the hexagon. So, she thought two hexagons express twelve.

113. S: perhaps, she made a new symbol, hexagon, for "6", because of 6. And as there are two hexagons and as 6 times 2 makes 12, so I think she made two hexagons.

After getting other answers (Δoo and oΔo) from the pupils, the teacher finished the lesson by saying "T: some people may still have difficulties in understanding the multiplication" and asking for bigger numbers to be represented, such as "100".

At this stage, I found in this class that the hypothetical rule "the composition of symbols represents the multiplication of correspondent numbers" expected by the teacher to be used in task 1 was sometimes not employed by some pupils. The answers with four triangles or two hexagons appeared from this reason. It means that there was no feedback of milieu for the answers given by these pupils. No use of this rule is directly related to the absence of the verification method which allows a feedback of milieu. Insofar the expected rule is used, the feedback will be given from by milieu. I found here the importance in the organisation of milieu by the teacher. However, I have to also mention the way of regulation or intervention in this lesson when the absence of feedback from some pupils is found by the teacher. It is not the teacher's direct intervention that gives feedback to the pupils. He only clarifies pupils' ideas and asks other pupils in the class whether they are correct or not and why. For example, the pupil's discourse [104] could be a feedback for the pupil who gave the wrong answer (ΔΔΔΔ). It's therefore the social interaction which allows feedback. This social interaction would not establish itself without the intervention of the teacher. The feedback was not given by the milieu itself but came from the social interaction enabled by the teacher. This is a negotiation of "didactical contract" trying not to owe all responsibility of validation to the teacher.

Concerning the targeted mathematical idea in this lesson (to view a number as a product of other numbers), as far as the anticipated rules – multiplication, correspondences between numbers and symbols – are employed, the pupils consider and use implicitly or explicitly this idea in order to find the symbols for the number "12". In particular, as the compositions of numbers' graphical representation explicitly shows, the pupils are aware of the idea clearly. In the last part of the activities for task 2, we can see that by asking the other way of expressing "12", the teacher tries to elicit an idea that the order of numbers in the product does not matter.

Discussion and Conclusion

What I analysed in this paper is only one case of many lessons given by a Japanese expert teacher. And this lesson was selected, due to the fact that some of the participants for the previous APEC specialist session in Tokyo will also be at this conference in Khon Kaen in Thailand. Therefore we cannot generate and conclude the results of analysis for all Japan. It is also true that some approaches well known in Japan – problem-solving oriented lesson, problem-discovery oriented lesson, etc. – might be more conform to the teaching and learning process indicated in the theory of didactical situations (see for example, *Japanese lesson study in mathematics at a glance* edited by Isoda et al.).

Let us come back to the initial question: *what are the characteristics of the "good" lesson or teaching in Japan?* As Stigler & Hiebert (1999) describe the Japanese lesson in the videotaped studies, "structured problem solving", the lesson analysed was organised so that the quite demanding problems are posed and the students invent their procedures or solutions. The teacher carefully designed and orchestrated the lesson. It seems that these aspects are recognised "good" part of Japanese mathematics teaching. Using the theory of didactical situations, we can explain them summarising in the following two points: the way of intervention of the teacher for the organisation or regulation of a class and the problem elaborated for the lesson.

For the first point, as we see in the analysis, the teacher quite rarely gives an answer or solution to the given task and he does not directly validate pupils' answer. He only asks the reason of a given answer (formulation), clarifies pupils' statements, and brings them to a common solution by respecting their ideas. Even though the pupil's milieu is not organised well enough to give feedback, it's not the teacher who gives feedback, but feedback from the other pupils is promoted by the teacher through social interactions. These actions, all have as a goal making a relevant didactical contract between the teacher and the pupils over mathematical knowledge.

For the second point, the teaching material elaborated for this lesson, in order for the target mathematical ideas (number as a product of other numbers and exception for some numbers) in the lesson to emerge. The graphical representation which allows to visualise the structure of numbers and number system is adopted. Furthermore, the problem, especially task 2, is set up so as to require these ideas as the means of establishing the optimal strategy to solve the problem or reach the goal of the "game". However, we have to also pay attention that this kind of teaching aid sometimes elicits a phenomenon called "metacognitive shift" in which the teaching aid becomes an object itself (Brousseau, 1997, pp.26-27).

As this lesson analysed in this paper was a part of lesson study which also demands criticism, this paper will end with a personal opinion taking into account the results of the analysis. As many factors are correlated to make one lesson, I could not propose a solution but mention just two points. First, it seems that the organisation of milieu for the pupils could be improved. In this lesson, even though the rule of multiplication plays a crucial role, multiplication itself does not receive a special status for some pupils more than addition. Due to this fact, some pupils use addition. As addition is more natural for people than multiplication for composed symbols (see the number systems developed in the world), it is necessary to elaborate a situation which allows pupils to give a special status to multiplication[4]. It seems that the way of questioning for task 1 was not clear enough to make some pupils find a hidden rule which govern the ordered cards and their continuation. Second, it seems that several situations, such as those of action, formulation, and validation were overlapped so much and could not take enough time to each situation.

Notes

[1] Lesson study is an approach of self training by in-service teachers for the improvement of teaching. It's very often practised in Japan. See for example, Stigler & Hiebert (1999).

[2] See the proceedings of APEC conference in Tokyo, for example, the paper presented by Inprasitha et al. (2006).

[3] It seems in the eyes of Japanese that the lesson analysed in the TIMSS video studies is rather "good" one.

[4] This is the opinion which participants in the discussion moment of this lesson also expressed.

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Mathematics Public Lesson Grade 4 Mathematics Instruction Plan

Teacher: Tsubota, Kozo, Vice-Principal, Tsukuba Fuzoku Elementary School

Research Theme: Examining instruction that focuses on “viewing a number in relationship to other numbers, such as a product of other numbers.”

1. Title Prime and composite numbers

2. About research theme

(1) Fostering rich sense of numbers

The current (2000) revision of the National Course of Study (2000) stresses that the goal of “fostering rich sense of numbers, quantities and geometric figures” is to be considered carefully. Since multiplication is introduced in Grade 2, a specific goal, “to view numbers as products of other numbers,” has been included. However, this is only one specific instance of developing “number sense” that must be addressed all the way though upper elementary school. Therefore, we must constantly address number sense intentionally. Today’s lesson proposes the treatment of numbers sense using the topic of “prime and composite numbers.”

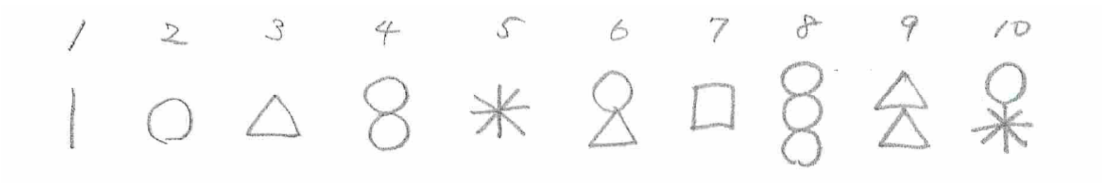
On p. 75 of Commentary on the Elementary School Mathematics Course of Study, you see a statement, “the goal is to develop an understanding of the multiplicative structure of numbers through an activity of counting objects by grouping.” Within the context of the introductory treatment of multiplication in Grade 2, this statement means that students should understand that a number can be viewed as a product of other numbers. For example, 12 can be thought of as 2 × 6 or 3 × 4.

In today’s lesson, we would like to further this perspective so that students can consider, for example,
12 as 2 × 3 × 3.

(2) Prime and composite numbers

In this lesson, we will pictorially represent the fact that all whole numbers are either prime numbers or composite numbers, which are products of prime numbers.

The following designs will be shown, and students are expected to identify rules the govern them. Then, using those rules, students will be developing designs for larger numbers.



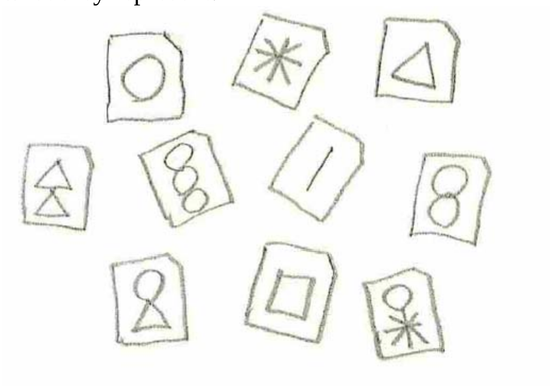
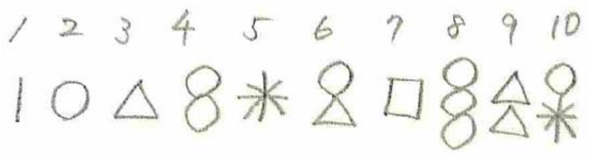
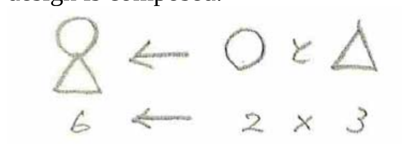
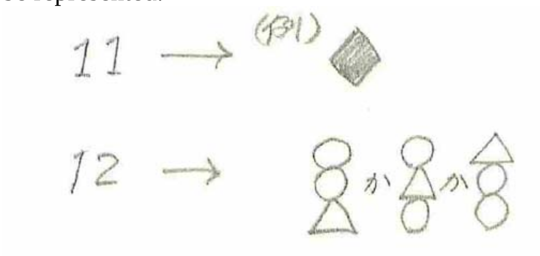
If students truly understand the ideas behind this lesson, they are more likely to understand the meanings of “least common multiples” and “greatest common divisors” to be studied in Grade 6.

3. Goals
To be able to view a number as a product of other numbers.

4. Instruction plan (2 lessons total)
   Understanding prime and composite numbers …… 1 lesson (this lesson)
   prime and composite numbers up to 100 ………… 1 lesson

5. Instruction of the lesson
   (1) Goals
   To notice that whole numbers are made up of prime numbers and their products.

   (2) Flow of the lesson

Instructional Activity	Points of Considerations
1. Observe the ten designs shown on cards and determine what they represent. 	(1) Post the ten cards on the blackboard at random. Ask students what they notice. ● If an idea that relates to numbers is raised, ask for the reasons.
2. Order the cards and identify "rules." 	(2) Guide the students to look at how the 6th design is composed. 
3. Using the discovered "rules," think how 11 and 12 can be represented. 	(3) Confirm that these designs represent numbers, then have them think about other numbers. ● Discuss and check the ideas for 11 and 12. ● Confirm that 11 must be represented by a new design while 12 can be represented by combining 2, 2, and 3.
4. Make a chart of number designs up to 20.	(4) Using the pattern they discovered, have students make the designs up to 20.

PROMOTING GOOD PRACTICES IN MATHEMATICS TEACHING THROUGH LESSON STUDY COLLABORATION

: A MALAYSIAN EXPERIENCE

LIM Chap Sam and CHIEW Chin Mon

School of Educational Studies,

University Sains Malaysia, Penang, Malaysia

cslim@usm.my

This paper begins by describing the elements of good practice in mathematics teaching as defined by the Malaysian mathematics teachers. An exemplar lesson plan collaboratively created by one of the Lesson Study group will be used to highlight the characteristics of good practice in mathematics teaching promoted through engaging teachers in Lesson Study process. Suggestions for adopting or adapting this lesson plan to another classroom context will also be discussed. A 10-minute video clip of an abstract of the lesson and some suggestions on how to use it for teacher professional development will be attached as Appendix to the paper.

Introduction

This paper aims to discuss and share our experiences of an attempt to promote good practices in mathematics teaching through Lesson Study collaboration project. First of all, we will describe the elements of good practice in mathematics teaching as defined by the Malaysian mathematics teachers. Then we will give a brief report of our Lesson Study project that aims to promote these good practices and discuss some challenges and issues that we have faced during the project. Next, to highlight the characteristics of good practice, an exemplar lesson plan collaboratively created by one of the Lesson Study group will be used. Finally, we give suggestions on how to adopt or adapt this lesson plan to another classroom context.

What is Good Practice in Mathematics Teaching?

As discussed in Lim (2006), generally, Malaysian practicing mathematics teachers agreed that a good lesson plan or good teaching practice should encompass the following characteristics:

a) student centered activities that encourage conceptual understanding

b) related to students' daily life experiences

c) that the students understand what is being taught and can apply what they have learned to solve problems

d) Good planning of student activities

e) Active participation of students in fun and meaningful activities

f) Use of teaching aids that enhance students' conceptual understanding

However, as experienced by many practicing teachers, these ideal characteristics are difficult to achieve in their day-to-day practice due to a number of challenges and constraints.

Challenges to Good Teaching Practices

a) Examination oriented culture

The Examination oriented culture is still very much prevalent in the Malaysian society. Examination results play an important role as a yard stick of accountability to the school performance. Hence, students' performance in examinations is used by school principals as a yard stick to evaluate teachers' teaching competency. Consequently, most teachers set their teaching priority on finishing the syllabus so as to ensure their students achieve excellent performance in these examinations. Very often, teachers need to spend a considerable amount of time conducting additional classes to prepare their students for these public examinations. This leaves teachers with very little time for professional development and innovative teaching.

b) Time constraint

Good practice of mathematics teaching requires student centered activities that promote conceptual understanding and active students participation. However, these activities are usually time-consuming. Due to the examination oriented culture as discussed above, teachers need to cover a fixed amount of syllabus within a limited teaching time. Hence, many teachers tend to adopt the traditional teacher centered approach that requires lesser preparation time.

c) Teacher's beliefs

Although most Malaysian teachers support the characteristics of good practice in mathematics teaching as mentioned above, many of them also believe that by giving clear explanation with suitable examples (teacher-centered approach) is more practical and good enough to achieve most of the teaching objectives. They feel that it is too much a hassle to allow students to construct their knowledge through student-based activities. They are not confident if their students could have acquired enough knowledge and skills by exploring the lesson themselves. Hence, the teachers tend to use the teacher centered approach where they can control the teaching and learning pace of their students.

Hence, due to the above challenges and constraints, teachers were shunning from innovative teaching approaches and continued to adopt the traditional teaching methods (Fatimah and Lim, 2004). As a result, there was limited time and opportunity for teachers to collaborate in school for the sake of their professional development. In other words, the teaching culture and school context failed to promote teachers to adopt innovative and good teaching strategies. Under such circumstances, it is a real challenge for many teachers to incorporate good teaching practice.

Lesson Study Project in Malaysia

In June 2004, we started a Lesson Study project in two secondary schools in Malaysia (see Lim, White & Chiew, 2005 and Chiew & Lim, 2005 for more details). The main aim of the project was to gauge if Lesson Study process could served as an alternative model for mathematics teacher professional development programme. Each school has 8 mathematics teachers participated. At the end of one year, one school has undergone three Lesson Study cycles but another school just two cycles. Nevertheless, both project schools received positively the Lesson Study model of teacher professional development, although one of the schools shows keener interest in implementing the project than the other. All the 16 participating mathematics teachers espouse positively that Lesson Study has (a) promotes a collaborative culture that enhances the professional collegial bonds within their mathematics staff; (b) gained and enhanced their mathematics content knowledge and pedagogical content knowledge through group discussion and peer observation; and (c) allow and encourage teachers to prepare better student based activities that constitute good practices of mathematics teaching and learning. In fact, even though our project has completed last June 2005, one of the project schools still persists with their Lesson Study group.

In view of the potential of Lesson Study collaboration for promoting good practice, we have set up Lesson Study groups in another two schools, one primary and one secondary school in January 2006.

The Chinese primary school is a small school with a total of 12 teaching staff and 6 classes. As 8 of them involve in the teaching of mathematics and English, they form a Lesson Study group. Their aim of setting up the Lesson Study group was to promote good practice in mathematics teaching as well as to enhance the teachers' confidence in teaching mathematics using English.

The secondary school is a fully residential school consists of selected students with above average ability. It has 10 mathematics teachers, so two Lesson Study groups were set up, with one for the upper secondary and one for the lower secondary mathematics. Their main aim of setting up the Lesson Study group was to promote good practice in mathematics teaching. They have set their first goal as promoting mathematical thinking and creativity among students.

Both schools started with a half day workshop which aims to introduce the concept and process of Lesson Study. The workshop was given by the researchers/authors, illustrated with a video tape entitled *Lesson Study: An Introduction* produced by Makoto Yoshida and Clea Fernandez of the Global Education Resources (2002). At the end of the workshop, the teachers form the Lesson Study group and began their first discussion by setting the goal and arranging schedule for the following meetings. Both schools faced similar problem when trying to arrange a teaching period that could be observed by all teachers and who would like to teach the lesson. All teachers were overloaded and had a busy working schedule, nonetheless, they managed to find a suitable time after much negotiation. Likewise, most teachers were shy and felt stressful to teach openly and to be observed by their colleagues. Perhaps, this is yet to be a culture of our Malaysian teachers. However, at the end, someone had to volunteer or was persuaded to be 'the teacher'.

After 3-5 discussions, all the three Lesson Study groups managed to have their first teaching observation. The teaching lesson was video-taped and reflection on the lesson was carried out immediately after the teaching. The lesson plan was then revised according to all participating teachers' comments and suggestions. Ideally, the revised lesson plan could be re-teach to another class. However, for the Chinese primary school, as it has only one class per grade level, it is not possible to re-teach the revised lesson to another class. For the secondary school, due to time and examination constraint, the teachers chose not to re-teach the lesson this year, but bring forward to re-use the lesson plan next year.

So far, the first two project schools produced 5 lesson plans while the last two schools produced 3 lesson plans. All were video-taped and analyzed. For the purpose of this paper, we will only discuss one lesson plan that best displayed good practice of teaching mathematics although we acknowledge that given time and effort, it could be revised and improved further. The VTR accompanied this paper is also based on this lesson plan.

An Exemplar Lesson

The chosen lesson plan was designed to introduce the concept of "set". Appendix 1 shows the complete lesson plan and the worksheet given. The target group was 20 Form 4 (Grade 10) students with above average ability. The lesson took 40 minutes to complete. The key mathematical concepts to be taught include: set, elements of a set, Venn diagram, number of elements, empty set and equal set. This is the first lesson on the topic of "set" for this group of students. However, they already have prior knowledge of "classifying things into collections" and "able to group objects based on certain common characteristics". The expected learning outcomes were at the end of this lesson, (i) the students were able to explain the concept of set to their peers; (ii) they can use the correct set notations such as braces { }, phrases and Venn diagram to represent a given set; (iii) they can identify the elements ($\in$) or non-elements ($\notin$) of a given set and its number (n); (iv) they can give examples of empty set ($\varnothing$) and equal sets.

- **Set induction**

The teacher started the lesson by asking his students where will they be going during their weekend outing. This is a fully residential school where all students are compulsory to stay in the school hostel. They were allowed to go outing only once a fortnight. The teacher knew that most likely they will visit the nearby hypermarket. The teacher then attempt to link today's topic by asking a few related questions such as "in a hypermarket, where do you find a pair of trousers? A tube of toothpaste?" etc.

To arouse the curiosity of the students, the teacher introduced a guessing game. The teacher asked a student, AA to pick up a red packet. Each packet contains a piece of paper written an amount of money and the name of an object to be bought. The teacher asked the class to guess what student AA was supposed to buy. The class could not answer, so the teacher gave a clue by asking student AA to go to a corner where he can find that object. At different corners of the classroom, there were labels such as 'Toiletries', 'Food', 'Clothes' and 'Books'. For example, student AA went to the "Food" corner. The teacher encouraged the students to guess the possible object that AA was looking for. Some students guessed the answer as "junk food"; "sweets" or "chocolate". Teacher then asked AA the amount of money given to buy. This provides another clue that narrows down the possible answers. After a few guesses, the students were able to guess the correct answer as "Maggi Mee".

The teacher repeated the game by asking a couple of students to choose the red packets again.

Comments:

This set induction fulfills/displays several characteristics of good practice:

(a) It links the topic to the students' daily life experience such as shopping at a hypermarket.

(b) The guessing game is fun and meaningful because it helps students to realize the importance of the concept of set and classification in daily life.

(c) By encouraging the students to guess, it promotes the creativity of students to generate a set of objects that share common characteristics which is the basic concept of set. By giving various clues or conditions, it encourages students' logical reasoning that helps to deduce the correct answer.

- **Setting the context**

After playing the guessing game, the teacher highlighted the importance of classification and organization in daily life. He then brought the students' attention to today's topic. He explained the definition of set, and pointed out the main concepts to be learnt today as well as the four activities to be played later. All these information were displayed on three manila cards placed on the blackboard.

This step is important because it aims to set the students' minds to focus on the learning objectives and the expected learning outcomes of today's lesson. This allows students to be ready and well prepared for the learning.

- **Learning by doing**

Instead of the usual teaching style of explaining the key concepts by giving examples, the teacher in this lesson has chosen to use the structures of cooperative learning. He planned out the following three activities to develop the lesson:

Activity (1) Fan and Pick

The teacher displayed 20 cards in the form of a fan and asked one member of each group to come to the front to pick 5 cards. Each card was written the name of an element, for example: 'January', 'March', '3', '2' etc. In each group, the students were asked to sort the 5 cards into different sets according to some common properties. Later, they were asked to compare and sort their cards with all the other three groups. They were then asked to paste all elements which share the common properties on the soft board at the back of the class. Earlier on, the teacher has drawn five oval shapes and labeled them as A to E. It was observed that all students participated actively and they managed to paste all the cards onto the relevant Venn diagram in less than a minute.

Based on the results of the activity, the teacher developed some key concepts of set such as 'representing a set in three ways – using phrases, Venn diagrams and braces { }'; empty set, element and non element of set and the number of elements. The students were seen to participate actively in the discussion and developing of the concepts together with their teacher.

Comments

The above activity was well planned and it highlighted several characteristics of good practice:

(a) The activity was student-centered where all students participated actively in sorting the given elements. The structure of the activity requested the students to help each other to get the task accomplished. This also encourages simultaneous interactions among students.

(b) Through effective questioning technique, the teacher was able to lead the students to participate actively in the discussion and to develop the conceptual understanding naturally.

Activity (2) Round table

The teacher gave each group a worksheet and a pen. Each group member took turn to answer the questions on the worksheet. The worksheet contains two parts. The first part asks the students to list the elements of each set A to E using the set notation. The second part gave a set P and asked students to determine whether a given element belong to set P. This exercise aims to assess students' understanding of concept.

To make the activity more fun and competitive, the students were encouraged to complete the worksheet in the shortest time. The worksheet of the first completed group was labelled 1 and subsequently for the other three groups. Each group then exchanged their answers for checking (pairs check).

Comments

This is again another well planned activity that displays some characteristics of good practice:

(a) it assesses if the students understand what is being taught and can apply what they have learned to solve problems.

(b) It promotes active participation of students in a fun and meaningful activity.

(c) Using the cooperative learning structure of 'round table', it encourages students to have equal participation. Every group member has an equal opportunity to complete the exercise.

However, it was observed that some group members who were more dominant tended to answer the questions individually. This observation was noted at the reflection of observing teachers and was suggested that teacher needs to be more alert when carrying out this activity in the future.

Activity (3) Mix and Match

This was an outdoor activity that aims to introduce the concept of 'equal set'. The teacher prepared 20 cards. Each card was written a set such as A= {1, 3, 5 }, C= {5, 1, 3}, E= {all positive odd number lesson than 7} or J= {s, u, k, a}, L={s, k, a, u}. The teacher threw all the cards into the air and each student picked one card when the cards fell to the ground. The students compared and matched their cards with their friends. Students who have cards which are of equal sets were asked to stand in a group. The teacher inspected each group to ensure that they have grouped themselves correctly. The process was repeated to give more practice to the students.

Comments

This activity is unusual because it brings students out of the classroom setting. All students were very excited and happy. Everyone was seen actively engage in the activity. Perhaps they perceived it as a game rather than a lesson. Hence, this activity displays several of the characteristics of good practice such as:

(a) it is a student centered activity that encourage conceptual understanding

(b) it encourages the student to learn and to apply what they have learnt

(c) It is a fun and meaningful activity that engage students actively in learning. The students were seen to yell and cheer while they learned happily.

- **Practice For Reinforcement Through The Activity "Think-Pair-Square"**

After the excitement of games and activities, the students were asked to do their work individually. They were given a worksheet that contains one question with 4 parts. The question asks the students to represent the given sets by Venn diagram as well as state the number of elements in each given set. When they have completed the questions individually, they were asked to pair with a group member to discuss and check their answers. If their answers were different, they were supposed to argue and to justify for the best answer. Finally all the group members were to make a final decision to accept the best and final answer for their group. The teacher then asked them to hand up their completed worksheets.

To further reinforce the students' skills, the teacher gave some home work exercise for the students by referring to the textbook.

Comments

This is a very common practice in Malaysian schools that mathematics teachers used to give class work and home work exercises that aim to reinforce the understanding of students at the end of the lesson. It is also a strong belief of "practice make perfect" that students need to drill and practice so as to master the skills that they have just leant.

However, in this lesson, the teacher has cleverly using another cooperative learning structure: 'think-pair-square' that not only encourages students' individual accountability but also encourages Vygotsky's principle of 'thinking & talking' in the process of learning to be applied here.

- **Closure**

The teacher asked one student to volunteer to recap what they have learnt today. The student was able to list out the key concepts learnt. The teacher then summarized today's lesson and emphasized the importance of learning set and set theory in daily life. He also referred students to other daily life examples such as finding a book in the library or searching for the address of a hotel in the telephone directory. The teacher then foreshadowed the forthcoming lesson to the students about some key concepts to be learnt in the next lesson such as intersection and union of sets.

Comments

The whole lesson took exactly 42 minutes. This shows that the teacher has managed the time very well. The closure was well done as the students were able to summarize what they have learnt in today's lesson confidently. The objectives of the lesson were seen to have been achieved.

Activity: Fan and pick

Activity: Mix and Match

Suggestions for Adoption/Adaptation of the Lesson Plan

After the lesson, all the mathematics teachers in the Lesson Study group sat down to discuss and reflect on the lesson. All the teachers agreed that it was a good lesson that depicted various characteristics of good practice in mathematics teaching and all of them would like to try out the various activities in their mathematics classes too. All the observing teachers also enjoyed the lesson as the students did. They commented that the presentation of the teacher was very clear and easily comprehensible. The activities were fun and meaningful.

However, there were 5 activities packed in one lesson. Even though the teacher managed to carry out all of them in the stipulated time, there was a bit of rushing and some observing teachers were worried that if all the students have managed to follow the activities positively. But the teacher in charge argued that it was deliberately planned this way so that the students will not get bored. These students are of above average ability. They can learn things very fast. They like to be challenged by a variety of activities. They get bored easily if the pace of the activity is too slow or not challenging enough for them.

Nevertheless, all the teachers agreed that the lesson plan can be modified to suit the needs and ability of students. For example,

a) For normal or lower ability students, the number of activity could be reduced. It is not necessary to pack all five activities at one time. Perhaps two to three activities might be enough to attract students' attention for learning.

b) For bigger class size such as more than 30 students in a class, the number of activity also should be reduced. This is because bigger class will have more groups; hence more time is needed for each group to present their answers.

c) The cooperative learning structure such as 'Round table', 'fan and pick', 'mix and match' can be used to develop different kinds of content or concepts learnt.

Lesson Study for Teacher Professional Development

After one cycle of Lesson Study, we encouraged the teachers to reflect and write down their reflection in a questionnaire provided. Analysis of the data show that most teachers perceived the Lesson Study process positively. They espoused that:

Lesson Study group

"good, it stimulates teacher to change the way of teaching in the class"

(upper secondary lady teacher)

"more input, more thinking"

(upper secondary man teacher)

"good, all mathematics teachers discuss together"

(lower secondary lady teacher)

"useful. Should be practice, enable teachers to exchange knowledge and experience, help teachers to overcome problems about lesson, enable teachers to discuss about lesson, many heads are better than 1. "

(lower secondary lady teacher)

The lesson study process has provided a meaningful experience for teachers to reflect on their own teaching while getting new ideas from their peers. We observed that when they discussed and collaborated in a professional manner, ideas of good teaching practices were examined through their self-reflection.

However, one teacher remarked that, "it is useful but time consuming" while another teacher found Lesson Study "must follow sequence and time frame". These comments are expected because the teachers were asked to make time and come together to discuss, at least twice or three times; then to observe the teaching, and to reflect. Due to some constraints in the school, teachers felt uneasy to juggle their time as they also have other teaching tasks and duties at the same time.

Concern over ‘time’ and heavy workload are prevalent and this would likely be the main issues, judging from the teachers’ responses about lesson study. However, we would like to argue this from a different perspective. Due to the recent trends and changes in the education, it is imperative that teachers change their mindset and be aware of their own professional development. Currently, we observed that teachers’ awareness of self-development in teaching is lacking in the school teaching culture. As such, we anticipate a long journey to promote teachers as life-long learner as demanded by the Malaysian Ministry of Education. In our view, Lesson study has provided an alternative and potential model of teacher professional development that deserved serious attention from the Malaysian educational authorities.

Conclusion

This paper has shown that lesson study process is able to disseminate the characteristics of good teaching practices through engaging teachers in a Lesson Study collaboration. More importantly, the positive and encouraging feedback from the participating teachers has motivated us to spread the lesson study project to more schools. However, we acknowledged that it is still early to make any conclusive findings based on the few lesson study conducted. To date, Lesson Study as a form of teacher-led professional development is still relatively new to the Malaysian teaching context. Implementation of lesson study projects will require the determination and support from the school administrators especially at the initial stage. However, we are optimist that more teachers will volunteer to participate in the lesson study process when they have realized the benefits that could be gained from lesson study process.

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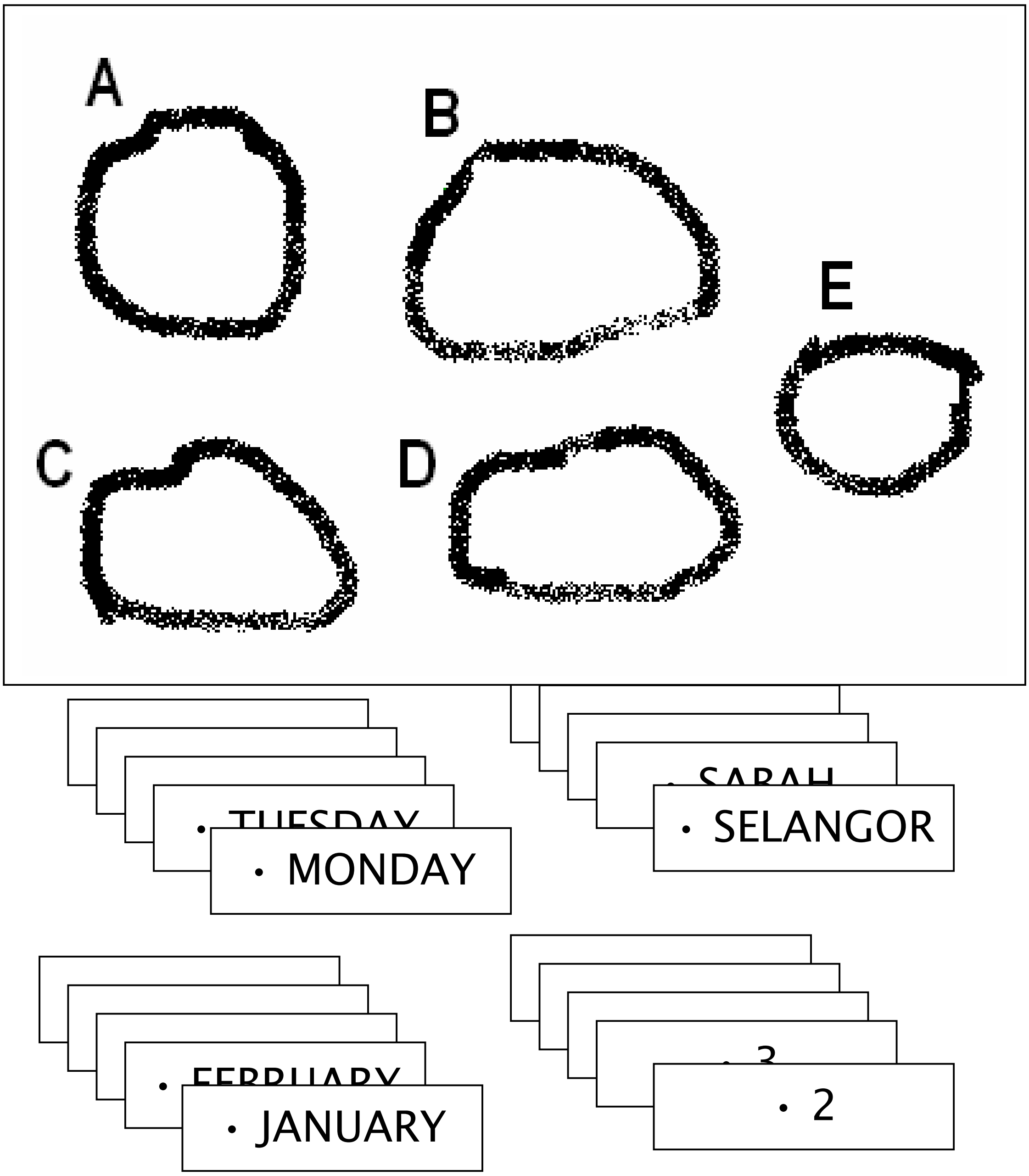
Appendix I: Lesson Plan

Date:	7 April 2006 (Friday)
Time:	9.50 – 10.25 am
Class:	Form 4S
Class Size:	20 students
Ability:	Above average
Topic:	SET (Form 4 or Grade 10 Mathematics)
Subtopic:	Understand the concept of set
Key Concept:	Set, elements of a set, set notations, Venn Diagrams, Number of elements, empty set, equal sets.
Prior Knowledge:	(a) The students have common sense of classifying things into collections. (b) The students can also group objects based on certain common characteristics.
Learning Outcomes:	(a) The students are able to explain the concept of set to their peers. (b) They are able to draw Venn Diagram and use the correct set notations. (c) They are able to identify equal sets.
Higher Order Thinking Skills:	Application, Analysis, Synthesis
Moral Values:	Being helpful and supportive.
Soft Skills:	Cooperation and teamwork.
Teaching & Learning Materials:	Worksheets, manila cards, double-sided tape, scissors, thumb tacks
Teaching & Learning Strategies:	Cooperative Learning (CL)- the *Structural Approach.
Classroom Setting:	4 groups with average 5 students per group

Procedure and Time	Content / Skill	Teaching- Learning Activities	Remark
1) **Set Induction** (5 min)	<u>Content:</u> Categorization/ classification <u>Skills</u> Critical & analytical thinking skill	Teacher begins the lesson by asking some daily life questions: In a hypermarket. Where do you buy a pair of trousers? A tube of toothpaste? A dozen of oranges? A packet of Maggi mee? A kg of tomatoes? **Guessing game** Teacher asks a student to pick an envelop containing a piece of paper written an amount of money and an object. The students need to go to the corner where he can find that object. Other students are asked to guess the object which that student is looking for. *(This will help to generate a set of objects having the same property.)* More real life examples: Where to find Science books in the library? (*Dewey Decimal System*) How to find the phone number of a hotel in the telephone directory? (*Alphabetical order*)	The classroom is label 'Toiletries', 'Food', 'Clothes', 'Books' at different corners. To help students to realize the importance of set theory in everyday life. Real life examples give them the significance of mathematics in everyday life.
2) **Setting Context** (2 minutes)	Overview of today's lesson	The teacher emphasizes the learning outcomes precisely and explicitly : (a) The students are going to learn the concept of set. (b) They will be able to draw the Venn Diagrams (c) The students will be able to identify equal sets.	Set the students' mind to focus on the learning objectives and the expected learning outcomes.

3) **Learning By Doing** (15 minutes)	<u>Content:</u> Set, elements of a set, set notations, Venn Diagrams, Number of elements, empty set, equal sets <u>Skills:</u> Analysis Synthesis Interpretation Presentation	Activity (1) CL Structure: **Fan & Pick** Each group is given 5 cards with an element on each card. They are required to discuss with other groups so as to form groups of elements with common properties. They are asked to paste all objects on the soft board according to similar properties. *(That will clearly show the Venn Diagram*) Refer to Appendix 1a Activity (2) CL Structure: **Round Table** Each group is given a worksheet and a pen. Each group member takes turn to answer the question, one by one. They compete between groups. They exchange the answers for checking (pairs check). Refer to *Appendix 1b.* Activity (3) CL Structure: **Mix and Match** The teacher throws pieces of cards with a set written on each of them. Each student has to pick one card and compare with their friends. Students having equal sets are asked to stand in a group. (This procedure may be repeated to give more practice to the students). Refer to *Appendix 1c.*	This CL structure encourages *simultaneous interactions.* Students need to help each other to get the task accomplished This CL structure encourages *Equal participation* Every student is taking part in the activity. CL structure: *Positive interdependence* This fulfils their excitement need. They will yell and cheer as they learn.
4) **Practice for Reinforcement** (10 minutes)	Reinforcement and Evaluation	Activity (4) CL Structure: **Think-Pair-Square** Students do their own work individually to encourage individual accountability. Then they pair up with a friend to discuss. Finally all members in the group make final decision to accept the final solution. The teacher asks them to pass up all their papers. Refer to *Appendix 1d.* Homework: 3.1 (a) – (d) for further reinforcement.	CL structure: *Individual Accountability* Vygotsky's principle of 'thinking & talking' in the process of learning applies here.
5) **Closure** (3 minutes)	Maximum Recall	The teacher recaps today's lesson by prompting the students to give the lesson's learning objectives. The teacher foreshadows the coming lesson to encourage the students to do their own reading when they go home.	The students recall and reinforce their learning. The students anticipate the upcoming learning topics

Appendix 1a: ***Fan and Pick***



Cards are shuffled so that the students will get the cards randomly. They are asked to sort them according to certain common properties and place them in the Venn Diagram above.

Appendix 1b: ***Round Table***

Topic: Set Group: ______

Structures: Roundtable

Instruction: Each member takes turn to answer the question, one at a time.

1) List the elements of the sets by using the set notation.

(a) A =

(b) B =

(c) C =

(d) D =

(e) E =

2) Given P = { all multiples of 5 from 20 to 40. }
= { }

Determine whether each of the following is an element of P by using the symbols $\in$ or $\notin$

(a) 3 $\square$ P

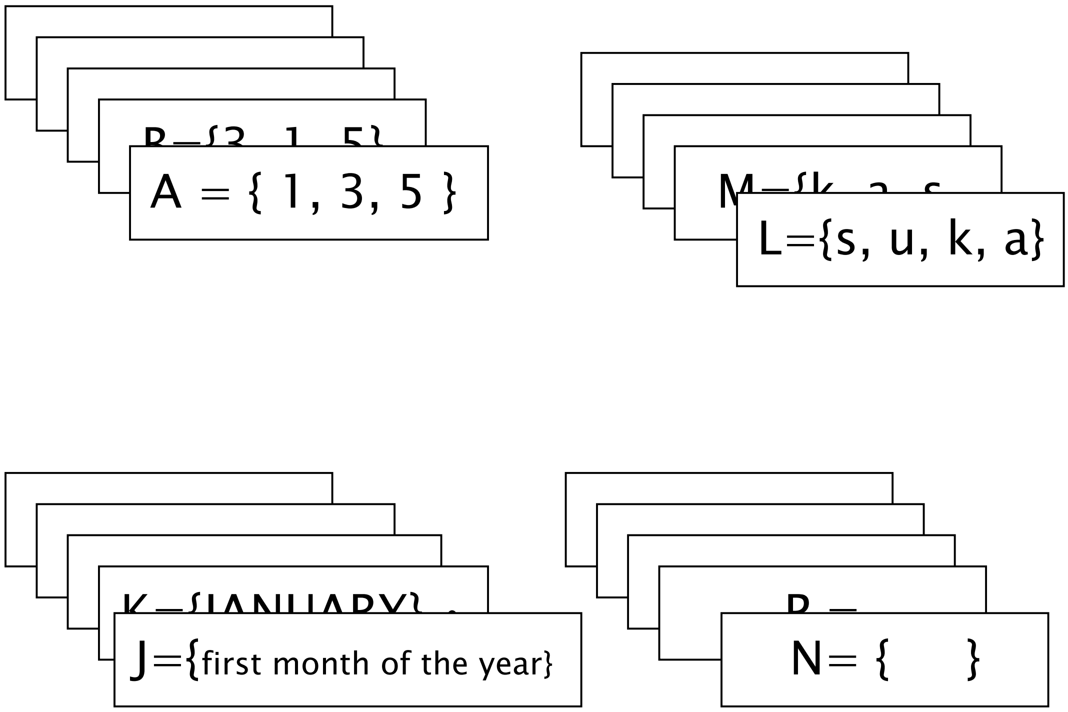
(b) 10 $\square$ P

(c) 25 $\square$ P

(d) 30 $\square$ P

Appendix 1c: ***Mix and Match***

The teacher shuffles all the cards. He throws it into the sky and the students pick one card when the cards fall to the ground. They match the sets on their cards and pair with others who have equal sets.



Appendix 1d: ***Think-Pair-Square***

Topic: Set Group: ______

Structures: Think-Pair-Square

Instruction: Complete the answers on your own. Compare with a friend. Check your answers in group of four.

1) Represent the following sets by Venn Diagrams and state the elements in each of the sets:

Set	Venn Diagrams
A = { banana, papaya, orange } n(A) =	
B = { 1, 3, 5, 7 } n(B) =	
C = { even numbers less than 15 } n(C) =	
D = { factors of 9 } n(B) =	

THE POTENTIAL OF LESSON STUDY IN ENABLING TEACHERS TO IMPLEMENT IN THEIR CLASSES WHAT THEY HAVE LEARNED FROM A TRAINING PROGRAM

Soledad A. Ulep

University of the Philippines National Institute for Science and

Mathematics Education Development (UP NISMED)

In the Philippines, there are different activities intended to help mathematics teachers grow professionally. Several of them have some of the characteristics of a lesson study but none has its full essence. This paper describes the possible ways by which lesson study in its pioneering stage in the Philippines enabled teachers to plan how good mathematics teaching practices to develop mathematical proficiency among students that they have learned from a teacher training program could be implemented in their own classrooms.

FORMS OF PROFESSIONAL DEVELOPMENT IN THE PHILIPPINES

According to Bentillo, et al (2003), the cascading model of teacher training is often used to implement changes on a nationwide scale such as the curriculum reform in the late 80's and the promotion of the practical work approach in the mid 90's. The training content is decided at the central level. The training moves from the national, regional, division, then school level with decreasing duration at each lower level. There is much dilution in using this top-down one-shot model. Another model called cluster-based training, involves teachers from several schools attending the same training program conducted by invited subject specialists as trainers. The content is determined by the master teachers of the schools in consultation with the teachers. While dilution may be avoided, the trainers may not be fully aware of the school situations so as to address training relevance. Recently, there has been an increase in the incidence of school-based training. This can be because of the recognition of the following: schools have specific teaching-and-learning needs that can be best addressed by the teachers of the schools working together, there are teachers in the schools who are capable of providing the training, the training can be done on a regular and continuing basis, and such training does not require much financial resources which the school can provide.

Besides training, curriculum materials development such as the ones under the Philippines-Australia Science and Mathematics Education Project (PASMEP) in the early 90's also helped teachers grow professionally. In groups, selected teachers from all the regions in the country who previously underwent training under PASMEP developed together daily lesson plans at UP NISMED with guidance from Australian and UP NISMED consultants. To try out for improvement, they demonstrated the lessons to other groups. They then went back to their classes to try them out with their students after which they came back to UP NISMED to revise and finalize the lessons accordingly. The process was done in three one-month curriculum-writing workshops with trying out in between. The outputs were two volumes each of daily lesson plans that were endorsed by the Department of Education for use by grades 8 and 9 mathematics teachers.

INTRODUCING MATHEMATICS LESSON STUDY IN THE PHILIPPINES

The researcher conceptualised introducing lesson study in order to enable teachers to use good mathematics teaching practices in their classes that would result to their students' mathematics proficiency. Several factors were considered to achieve this. First, the teachers who would comprise the lesson study group need to see good mathematics teaching practices “in action” so that they would get a very clear idea of what they are. As earlier studies reveal, teachers mainly teach by exposition (Department of Education, et al 2000). They first provide the definitions of terms then present the rules/ procedures and apply them using several examples. After which, they ask students to practice the skills that they have learned by doing several exercises. They present problems that are often worded at the end of the treatment of the topic when the students already know the procedures to deal with them. Thus, these problems are just routine ones and are often with only one method of solving. So the teachers who will be in the lesson study group need to know that good teaching practices involve among others: raising questions that give opportunities for all students to contribute an answer, making students think, providing problems/questions that may have many different ways of solving and/or may have many different correct answers, using real-life situations whenever possible and relevant, developing mathematics concepts, ideas, and skills based on problems (that is, teaching mathematics through problem solving), building on students' previous knowledge and experiences, requiring students to argue clearly and convincingly about the correctness of their answers, and making available follow up tasks to reinforce what students have learned. The above list of good teaching practices is based on the outputs of the workshops of the specialists’ sessions of the APEC Conference on Innovations in Teaching and Learning Mathematics held in Tokyo on January 15 – 20, 2006.

Second, the teachers need to be willing to perform the tasks involved in the lesson study. This willingness might stem from their open-mindedness and desire to develop professionally. Moreover, their administrators have to provide the needed support to make lesson study work. "Which school and who among the teachers in the school would be willing to venture into lesson study?" was the big question that confronted the researcher. What she considered very important all along was that there has to be a context for introducing lesson study. Thirdly, lesson study should fit naturally into the teachers' overall school activity so that they could do it easily. Lastly, the researcher realized that conducting a lesson study without exposing teachers to good teaching practices would yield a lesson study that has no substance and modelling good teaching practices without conducting a lesson study would not promote continuous professional growth.

The Training for Pasig City Secondary Schools Mathematics Teachers

The researcher had to look for the appropriate opportunity and time to introduce lesson study. The months of February and March 2006 were inappropriate since these were the last months of the school year. Teachers were very busy finishing their lessons. However in March, there was a request for the Mathematics Group of UP NISMED where the researcher is a member, to conduct trainings for the secondary school mathematics teachers of Pasig City in all the four-year levels. The trainings would be in April when it was already school vacation time. The trainings that were cluster-based and included 10 schools were held at Rizal High School, a school with about 9000 students, which was in the cluster. The researcher realized that the trainings could take into account the factors that she considered. They could provide the context for showing to the teachers what good mathematics teaching practices are so that they can reconceptualize what it means to be good mathematics teachers. They could also naturally provide the rationale for engaging in lesson study.

A request to allow four grade 8 mathematics teachers to work with the researcher to conduct a lesson study was sought from the division schools superintendent and the principal of Rizal High School. There were 11 teachers from the school. Since the school had previously participated in the international research Learner's Perspective Study (LPS) in which the researcher was also involved, the principal granted the request and assigned the department head to coordinate with the researcher. The department head's involvement was beneficial because she provided the needed support. The researcher chose one of the teachers previously considered for the LPS while the department head chose the other three teachers. These were the better teachers in the school. The researcher thought that if they could be exposed to the process of lesson study, they could comprise the core group that can introduce it later to their fellow teachers.

The training for grade 8 teachers was conducted on April 24 to 28, 2006 from 8:00 a.m. to 12:00 noon and 1:00 p.m. to 5:00 p.m. There were sessions on sample teaching that were problem-based, emphasized connecting concepts and procedures, and making sense of mathematics, highlighted mathematical habits and higher order thinking skills, and exemplified assessment as an integral part of teaching. To some extent, the sessions also attempted to address teachers' beliefs and practices.

Orientations About the Lesson Study

All meetings with the researcher related to lesson study were done after the sessions ended at 5:00 p.m. On the first day, the department head, two teachers, and the researcher met. The researcher asked them what kind of students they envision to have as a result of having gone through grade 8 mathematics. She also asked what a teacher's role is to achieve such a vision. She then described briefly what a lesson study is. According to the teachers, they envisioned that their students would know how and where to use or apply what they have learned and that they would develop logical thinking and discipline. They claimed that students' retention depended on how teachers presented the lesson. A teacher said that she encouraged students to solve a problem in different ways and she was surprised that at times they preferred their peers' solutions than what she offered because the former were easier for them to understand. She required students to explain their solutions/answers and not just to read them. From these accounts, it can be inferred that the teachers' ideas of good mathematics teaching though limited, were aligned with the framework of this project.

On the third day, the department head, the four teachers, and the researcher now a complete group, met. There was further discussion on the lesson study. The researcher lent the CD on lesson study developed by Global Education Resources (2002). On the fourth day, the lesson study group under the leadership of the department head and without the researcher, met to clarify the teachers' involvement in the professional development activity. During their break on the fifth day, the group without the researcher listened to the CD. After the training session that day, the researcher asked the kinds of professional development activities that the teachers engage in. According to them, they have a monthly in-service "trainings" that are planned a week before the school year begins. Each training which is done from 2:30 p.m. to 5:30 p.m. every third Thursday of the month when all classes are over, is of two types: demonstration teaching and reporting or sharing. If there is something new to be shared such an innovative strategy for teaching a topic, a teacher is assigned by the department head to prepare the lesson plan for it which she checks. The teacher may consult other teachers in preparing the lesson. During the demonstration teaching involving the teacher's actual class, the teachers who observe may come from all year levels. They see and get a copy of the lesson plan only on that day that the lesson will be carried out. After the demonstration teaching, a discussion follows in which the teachers discuss the results of the observation checklist that they accomplish while they observe the class. However, there is no documentation of the improved plan if at all it is revised and copies are not given to teachers. In short, there is no systematic and comprehensive collaboration among teachers in the development of lessons. Actual classroom results when the lessons are carried out are not documented. Apparently, there is no intention to document the suggestions for improvement and incorporate them in the plan and have the modified plan accessible to other teachers.

The teachers who attend a training/seminar are asked to report to the other teachers what they have learned and to share with them the handouts they obtained from it. Based on the results of national and regional student achievement tests, the teachers identify the least learned competencies. The department head would then assign some teachers to discuss the problematic topics so that other teachers can teach them well to their students. Such is another form of sharing.

According to the members of the lesson study group, the teachers prepare their lessons individually seeking help from others only as they need it. The lesson plans are skimpy. They do not provide the necessary details on the questions that the teacher will raise to develop concepts and the anticipated variety of responses from the students. As such they do not make it easy and natural for the teacher to develop students' thinking based on the kind of responses that they give. Oftentimes, questions are not also those that call for many different correct answers.

Hence, it may be said that the teachers to some extent help one another in preparing lessons giving the activity some form of collaboration. However, they do not come up with collaboratively and carefully developed lessons that are well-documented and which detail exactly the activities that the teacher and students will engage in as well as the questions that the teacher will ask and the answers that the students are expected to give. They also do not include other remarks that will guide the teacher to teach effectively. None of those that the teachers have done before has the real essence of a lesson study.

USING LESSON STUDY TO PREPARE FOR THE CLASSROOM IMPLEMENTATION OF LEARNINGS FROM A TEACHER TRAINING PROGRAM

During the last training day, the group met to decide on what topic to do lesson study. The first mathematics topic in grade 8 for the school year is systems of linear equations and inequalities (Department of Education 2002). The teachers admitted that it is difficult for many students. In the training, there was a sample teaching on "Linking Concepts and Procedures: Systems of Linear Equations." The teachers agreed to collaboratively develop a lesson plan about systems of linear equations in two variables based on how they understood and experienced the way it was presented to them in the training.

The researcher first asked the teachers how they teach the topic. One teacher said that first, she defines what a system of linear equations is. Then each day for several days, she teaches the procedures for solving systems using the substitution method, graphical method, and elimination method each time highlighting the disadvantage of a method to provide the need for other methods. Lastly, she gives word problems that involve solving systems for which students can use any method. She reasoned out that she teaches this way because she follows the sequence of the competencies listed in the Basic Education Curriculum (BEC).

Since the training was short, there was very little provision for teachers to reflectively discuss about their current teaching practices, about how they view what they learned in light of what they have been doing, and about how they intend to make use of what they learned in their own classroom teaching. In the training, they have been introduced to new ideas and have been made to experience teaching approaches that were learner-centred such as actively engaging learners in constructing mathematical knowledge. They wanted to find out if these would work in their actual classroom contexts.

The teacher who was chosen to carry out the lesson that the group prepared together, expressed that she appreciated how "systems of linear equations in two variables" was developed in the training. Starting from a single simple real-life situation, many mathematical ideas emerged towards the end of the lesson such as the meaning of systems of linear equations. The graphical and substitution methods of solving systems were naturally put to the fore from considering the situation. Such a reaction which the others in the group shared implies that the teachers realized that the way they teach the topic may still be improved; that for as long as the topics are covered, the sequence of presentation does not have to be as they are ordered in the BEC; and that it does not mean that only a single competency needs to be taken up each day. However, they raised concerns on where they can break the lesson as it was presented in the training, in order to give place to the practice exercises that will reinforce the new concepts and skills that the students will learn. They were also concerned on how they can give daily end-of-the-lesson-evaluation that they have traditionally been doing to find out if students have mastered the lesson for the day if they adopt the approach they encountered in the training. They asked if there is really a need for them to continue administering this evaluation on a daily basis.

The implementation of the lesson plan that the teachers developed together will be in the first week of June 2006 when classes resume. So its description and that of the discussion after the lesson is taught cannot be included anymore for the purposes of this paper. Nevertheless, during the paper presentation, a video of the classroom teaching and a discussion relating to it will be taken up.

Some Comments on the Lesson Plan

The teachers will meet again the week before classes start on June 5 to improve the plan shown on the Appendix. As it is, the lesson encourages maximum participation from the students right at the very start. Question 1 is easy enough for everyone to be able to contribute an answer. In Question 2, the students can draw upon their experiences for the answer because the situation is based on real life. However, after asking Question 3, students should be asked to compare their estimated answer to Question 1 and their answer to Question 3 to determine how good their estimates are. Question 4 might have been intended to make students realize that there can be many different correct answers. It provides a concrete meaning to the mathematical concept that an infinite number of ordered pairs can satisfy a linear equation in two variables. Given the real-life context, it means that several discrete values of the two quantities satisfy the given condition. In the classroom, it may be anticipated that different students may give different pairs of values. What the teachers had done was to summarize those possible answers sequentially using a table and labelled the two quantities x and y. What they missed was to explicitly write in the plan a question that will require the students to explain how they would arrive at those pairs of values. In Questions 5 and 6, the teachers apparently had included both the correct and incorrect answers that students may give. In their discussion on Question 5 while planning the lesson, they pointed out that there are equivalent correct equations. What they need to explicitly state is how they would process the wrong responses and what they would do so that students would recognize the equivalence of the different forms of correct equations that they have given. It was only after Question 6 where the teachers would introduce the meaning of a system of linear equations (although they had not written the formal definition) and it came out very logically and naturally in the flow of the lesson. The teachers appreciated this approach. It is definitely in contrast with what they had always done.

Further along the lesson, the teachers must have wanted the students to understand what the common values that will satisfy both of the two linear equations in the system would mean graphically. However, they should have provided the expected interpretations. Lastly, substitution as one of the methods of solving a system of linear equations in two variables was naturally introduced and practice exercises were provided later. Another real-life situation was presented as a context for the application of what students have learned. Again, the teachers need to give the correct answers to the questions raised.

To sum up, by continuously raising appropriate questions, the teachers aimed at actively involving students in generating mathematical ideas. In particular, they would teach mathematics through and for problem solving. Apparently, the way the teachers planned to carry out the lesson with their students showed that they had deliberate attempts to try out what they had learned from the training program.

CONCLUSIONS

The purpose of conducting a lesson study after the teachers had participated in a training program was to ensure that they are adequately prepared to implement the good teaching practices that were modelled in which they have experienced learner-centred teaching strategies. So far, only the planning stage in the lesson study cycle was reached as of this writing. Even so, it had already provided them the important opportunity to collectively and systematically reflect on their classroom practices. It was during this stage that they verbalized their realization that their teaching of a specific topic can still be improved, that there are concerns that they need to address in the process of making changes for improvement, and that the bases and reasons for their long-held practices have to be examined. It was also then when they substantially shared to each other their experiences in teaching the topic and worked collaboratively in preparing the lesson from start to end. As Bell and Gilbert (1996) note, when teachers have focused interactions about what they have learned and planned together on how they could adapt them in their own classes, the learning becomes clearer to them. Lastly, it was also then when they have initially put to action their willingness to try out in their classes the new ideas and approaches that they have encountered for the first time in the training. Although they may not be aware of it, the teachers have somehow grown personally, socially, and professionally in the process (Bell and Gilbert 1996). Moreover, they have begun to engage in the study of their own practices which is one characteristic of successful professional development programs (Glickman, et al 2001). Studies show that after undergoing training, teachers often revert to their usual classroom practices such that innovations sometimes do not get implemented (Talisayon, et al 2000). However, in the case presented here, there are good indications that lesson study had enabled the teachers to be prepared to implement the innovations that they had learned.

RECOMMENDATIONS

The next lesson study meeting will be held during the mathematics department's planning meeting for the whole school year. The teachers will have to improve the lesson plan to fill in some gaps and systematically address their concerns. Also they need to plan for which other topics they have to develop lessons together. They can adapt those that were covered in the training. Then they can attempt to develop their own original lessons. If they have already internalised what good mathematics teaching practices are, then they should be able to exhibit them in their classroom teaching. They can involve the other teachers in the department. They can also learn how to make extensive documentations of the accomplishments of their lesson study group and make them accessible to other teachers through publications or presentations in workshops and conferences.

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LESSON PLAN

(as of April 28, 2006)

Topic: **System of Linear Equations in Two Variables**

(to be covered for several days)

Objectives: At the end of the lesson, the students should be able to:

1. formulate equations representing mathematical situations
2. state the meaning of a system of a linear equation in two variables
3. solve problems involving systems of linear equations in two variables using different methods

Materials: mangoes, oranges, graphing board

Prerequisite Knowledge and Skills: linear equations in two variables, graphing on the Cartesian plane

Instructional Procedures:

1. Show to the class 1 piece of mango and 1 piece of orange taken from a plastic bag of mangoes and oranges.

Ask: Which do you think is heavier? (**Question 1**)

Expected answers:

1. *The mango because it is bigger.*
2. *Students will heft the fruits first before answering.*

Ask: What do you think is the weight of this mango? this orange? (**Question 2**)

Expected answer:
Based on their experience, students can estimate the weight of each fruit.

2. Present the following information: One kilogram of mangoes consists of 4 pieces of mangoes and 1 kilogram of oranges consists of 5 pieces of oranges provided each fruit of the same kind weighs the same. (**Information 1**)

Ask: What is the weight of each mango and each orange? (**Question 3**)

Expected answer:

The weight of each mango is 250 g and each orange is 200 g.

3. Ask: If this bag contains 6 kg of fruits (mangoes and oranges), how many of each kind are there? (**Information 2, Question 4**)

Expected answer: (Ordered pairs given will be organized into a table like the one below later and the two quantities will be represented using variables)

Number of mangoes (x)		*20*	*16*	*12*	*8*	*4*
Number of oranges (y)	*5*	*10*	*15*	*20*	*25*	

4. Ask: Can you make an equation out of this table? (**Question 5**)

Expected answers:

$4x + 5y = 6$

$250x + 200y = 6\ 000$

$y = \frac{-5}{4}x + 30$

$y = \frac{5}{4}x + 20$

$\frac{x}{4} + \frac{y}{5} = 6$

$5x + 4y = 120$

5. Present the following information: Suppose the number of mangoes is 4 times the number of oranges. Can you write an equation for this? (**Information 3**)

Expected answers:

$x = 4y$

$y = 4x$

6. Say: So the two equations we have based on the given information are:

$$5x + 4y = 120 \qquad \text{Equation 1}$$

$$x = 4y \qquad \text{Equation 2}$$

Since x represents the number of mangoes and y represents the number of oranges in both equations, then we should have the same value for x and the same value for y in both equations. So we will solve these equations simultaneously. Together, the two equations that we are solving simultaneously are called a system of linear equations in two variables. The solution satisfies both equations.

7. Let the students graph the two given equations. Let them describe/interpret the graphs.

Expected answer:

The lines representing the two equations intersect or they have a common point. The coordinates of this intersection point are the values of x and y that are common to the two equations. They satisfy both equations.

8. Say: Examine the two equations:

$$5x + 4y = 120 \quad \text{Equation 1}$$
$$x = 4y \quad \text{Equation 2}$$

Since the value of x and y are the same in both equations, then we can replace 4y by x in the first equation. This gives:

$$5x + x = 120$$
$$6x = 120$$
$$x = 20$$

Solving for y using equation 2 since it is simpler, we get

$$20 = 4y$$
$$5 = y$$

So the solution of the system is (20,5). The method that we used to solve the system is known as the substitution method. This is one of the methods used in solving systems of linear equations in 2 variables.

9. To ensure that the students understand the substitution method of solving systems of linear equation in 2 variables, let them solve the following systems.

a. $x + y = -12$ b. $3x + 2y = 8$

$y = 3x$ $x = 2y$

Expected answers:

Solution: (-3, -9) *Solution: (2, 1)*

10. For further application of what they have learned, give them the following problem:

Michael left his home one morning to jog. At the same time, Sara whose home is 1 km away from Michael's, also left for brisk walking. Suppose Michael jogged at 6 km per hour and Sara walked at 3 km per hour, both at about a constant speed.

a. Use equations or graphs to show the distance-time relationship for each person.

b. What information can we get from the graphs/equations?

Expected Responses:

1. *The students might ask the following questions:*
    a. *Are they heading on the same direction?*
    b. *Are they heading on opposite directions?*
    c. *Are they heading towards each other?*

2. The resulting graphs are 2 lines that intersect. As such, the students might assume that Sara and Michael will meet.

Note: If time permits, give the above problem to be answered in groups. If not, it will serve as their assignment.

Prepared by: Revie G. Santos, Francisca R. Unida, Mylene B. Opeña, and Reynaldo R. Salamat Jr.

DESCRIPTIONS OF THE VTR ON THE LESSON “DEVELOPING THE MEANING OF A SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES”

(Based on the actual teaching of the lesson on June 8, 2006)

Summary

“Developing the Meaning of a System of Linear Equations in Two Variables” is the topic of this second year (grade 8) mathematics lesson. The objective is for the students to formulate equations representing mathematical situations, and to determine how the solutions of one equation may be related to those of the other.

Components of the Lesson

1. The teacher showed to the class a mango and an orange taken from an opaque bag. She asked the students which is heavier (**Question 1**) and to estimate their weights (**Question 2**). They were able to answer the first question correctly. However, their estimates revealed that they were not good at estimating weights. This valuable finding would not be obtained if she used exposition. Using learner-centered teaching strategies made it possible for students' weaknesses to surface. After this introductory activity, she presented the first information shown below.

One kilogram of mango consists of 4 pieces of mangoes and one kilogram of oranges consists of 5 pieces of oranges provided each fruit of the same kind weighs the same. (**Information 1**) What is the weight of each mango? each orange? (**Question 3**)

2. After they correctly answered 250g (or $\frac{1}{4}$ kg) and 200g (or $\frac{1}{5}$ kg), respectively, she presented the second information that follows.

If this bag contains 6 kg of fruits consisting of mangoes and oranges, how many of each kind are there? (**Information 2**, **Question 4**)

She asked them to work in small groups and to come up with possible values. She called on individuals, each time writing the phrases “number of mangoes” and “number of oranges” as she got a value for each quantity. Later, she asked how their data could be organized and how the quantities could be represented. They correctly answered “make a table of values” and “use variables” (x for the number of mangoes and y for the number of oranges), respectively. After the table was set up, she asked if it was possible that there was only one kind of fruit in the bag. A student responded that it could not be because of the given information.

Through the question, she made them realize that many different values satisfied a given condition but there were also values that did not because of the given context. Also, in order to communicate information concisely and efficiently, she led them to organize data using a table and to represent changing quantities using variables.

3. Later, she asked them if they could make an equation based on the table of values. They gave

the following equations which she wrote on the board: $\frac{1}{4}x + \frac{1}{5}y = 6$, $4x + 5y = 120$, $4x + 5y -$

$120 = 0$, $y = -\frac{4}{5}x + 24$, and $x = -\frac{5}{4}y + 30$. She later asked if all of them were correct and how

one could know which ones were correct. A student answered that if the equation became true when ordered pairs from the table are substituted then it is correct. Some students who verified their answer asked her to disregard it specifically, $4x + 5y = 120$, $4x + 5y - 120 = 0$, and $y =$

$-\frac{4}{5}x + 24$, even before the class verified it. She asked them to explain why they considered it

wrong.

While verifying if $x = -\frac{5}{4}y + 30$ was correct, a student asked if she could give another equation. The teacher said that she would call her later. After the answer was determined, she called the student who gave $5x + 4y = 120$ which the class accepted as correct. So eventually only the two equations $\frac{x}{4} + \frac{y}{5} = 6$ and $5x + 4y = 120$ were left. She asked how it was possible that there were two equations that represented the same situation and if they were related. A student answered that their solutions are the same and so they are equivalent. Another student explained how $5x + 4y = 120$ could be obtained from $\frac{x}{4} + \frac{y}{5} = 6$ by applying the properties of equality. She later asked them which equation they preferred and why.

With the pleasant manner that she handled the wrong equations that they gave, she gave the impression that it was alright to make mistakes. It was also good that she called on the student who volunteered to give an additional equation. More importantly, she used it as an opportunity to call the students' attention regarding the relationship of the two different-looking correct equations given. Asking them which equation they preferred made them aware that while different answers may all be correct and acceptable, one may be preferable because it is easier to use.

4. Then she presented the third information shown below.

Suppose the number of mangoes is 4 times the number of oranges. (**Information 3**) Can you write an equation for this? (**Question 5**)

They gave the following equations: x = 4y, y = 4x, y = $-\frac{x}{4}$, and 4x = y where x represents the number of mangoes and y represents the number of oranges. She asked if all the equations were correct and how they could tell. A student said that values from the table should be substituted to the equations. She asked if any ordered pair from the table could be used. Then there was an exchange of ideas among a few students while the rest of the class keenly followed. Whenever she directed a question to the entire class, the students immediately responded in chorus. In particular, she asked them to analyze if the ordered pair (8, 20) that is found in the first table satisfied the third information.

According to Christian they should make another table. He gave the following ordered pairs: (20,5), (16,4), (24,6), and (32,8). Interestingly, these values corresponded to the equation x = 4y and not to the equation y = 4x that he gave earlier. The teacher did not catch this. When she asked if what he gave were possible values for the information, Carol disagreed. She said that (16, 4) was not equal to (16, 10), apparently thinking that the values that satisfy the third information should also satisfy the second information. The teacher noted that Carol was already relating the two tables. But Christian said that the two tables should not be related. According to him, if there was a new problem, there would be a new equation and so there should be a new table of values. The teacher asked the class if they agreed. They did not. When she asked for more observations and reactions, Mutya asked “Magkarugtong ba ito?” “Is this (referring to the third information) a continuation of the one before it?” (referring to the second information). The class answered “Yes.” The teacher recounted how the presentation of information about the fruits in the bag progressed. So when she asked if the new table was related to the previous table, the class said “yes” and Christian said that he was changing his answer.

Christian’s point seemed to be that the first table corresponded to the equation 5x + 4y = 120 which was based on the second information. The second table which he gave corresponded to the equation x = 4y which was based on the third information. But instead of using the word “information”, he used the word “problem” so he considered the two information as two different “problems.” Meanwhile the teacher and the rest of the class seemed to consider the entire situation that included all the information as one problem. Up to the part that he thought that there should be a separate table of values that corresponds to each equation that is based on a “problem”, he was correct. What he needed to see was that afterwards, the relationship between these tables of values had to be determined. He later realized that the first table was related to the second table. He said that all the values of the second table satisfied the second equation but there were values in the first table that also satisfied the second equation. He must be referring to (20,5) which is only one ordered pair.

The teacher brought the class back to checking which of the equations they gave for the third information was correct. Aries said that his equation y = $-\frac{x}{4}$ was incorrect. It should be y =

$\frac{x}{4}$. Yves said that only the first equation, $x = 4y$, was correct based on the third information. The teacher asked if $x = 4y$ is related to $y = \frac{x}{4}$. The students recognized that they were equivalent but that they preferred $x = 4y$ because it was not in fraction form.

In the process of establishing that the second information was related to the third information, and so their corresponding tables of values were also related, and thus, their associated equations were likewise related, the teacher welcomed students' viewpoints. She gave them the opportunity to discuss them. Carol had noticed that (16,10) and (16,4) were ordered pairs that satisfied only one but not both equations. It was possible that just like Christian she noticed that (20,5) satisfied both. Though she could not elaborate on her answer, he tried to argue convincingly.

5. The teacher wrote on the board the two equations that they finally have: $5x + 4y = 120$ and $x = 4y$. By asking what the variables in each equation mean, she led the class to realize that their values are the same for both equations. So, she explained that they will solve these equations simultaneously. She added that the two equations that they will solve simultaneously illustrate a system of linear equations in two variables and its solution satisfies both equations. Hence, it was only at this point that she introduced the meaning of a system of linear equations.

Possible Issues for Discussion and Reflection with Teachers Observing the Lesson

- What good teaching practices did the teacher exhibit in the lesson?
  - She raised a question that gave opportunity for all students, regardless of ability to contribute an answer.
  - She used a real-life situation as a basis for introducing a mathematical problem
  - She asked students to estimate.
  - She developed mathematics concepts and skills through problem solving. In short, she taught mathematics through problem solving.
  - She asked students to discuss in groups to determine the possible answers to a question.
  - She built on students' previous knowledge, skills, and experiences.
  - She challenged students how they knew if their answers were correct and made them evaluate which correct answer they preferred and give reasons.
  - She wrote all the student responses to questions, both correct and incorrect, and provided them the opportunity to discover and explain the reasons for their incorrect ones.
  - She required the students to argue clearly and convincingly about the correctness of their answers and did not interfere with what they explained.
  - She consolidated important parts of the lesson for students to realize and appreciate connections or relationships.
  - She accommodated students' questions and additional answers even when the lesson had already progressed beyond the question for which the answer was given.
  - She asked questions that had many different ways of finding the answers and many different correct answers.

- She made students think.
- She made students realize or consider connections among their seemingly different responses.
- She fostered a very friendly classroom atmosphere that encouraged students to answer her questions and even volunteer them, and also to raise questions without fear that their answers may be wrong or their questions may not be appropriate.

• What else could the teacher have done to make her teaching effective?

  - She could have asked the students how many pieces of mangoes (or oranges) of the size that she had shown there are when they or their mother buy a kilogram of these fruits. Based on their answer, they could determine how good their estimated weights of the fruits were.
  - She did well to accommodate the seemingly different or conflicting answers of the students. However, she needed to be more careful in analyzing their responses. For example, she should have asked Christian to identify or write the second equation that he referred to when he said that all the values in the second table that he gave satisfied the second equation. This she should have done to make sure that everyone correctly understood what he explained. His table of values was correct. However, the equation that he gave for the third information was incorrect. Nonetheless, his idea that to a given information there is a corresponding equation and table of values, remained correct. It was possible that based on the table that he gave, the second equation that he meant was not the second one listed for information 3 but the first one listed for information 3. This is if he already realized that the equation that he gave was wrong. It was second in the sense that $5x + 4y = 120$ was the first and $x = 4y$ was the second. Moreover, she should have asked him to specify the values in both tables which he claimed satisfied both equations.

• What was the significance of the question of Mutya "Magkadugtong ba sila?" [meaning "Is this (referring to the third information) a continuation of the one before it?" (referring to the second information)] . Explain your answer.

  - This question made the class focus on a very important matter – that ultimately they have to find an ordered pair that would satisfy the two equations $5x + 4y = 120$ and $x = 4y$. This is what the lesson is all about.

• According to Christian, given a "problem" (meaning information), there is an equation and a table of values associated with it. All the values satisfy that equation. However, there are values from another table associated with another "problem" (meaning another information) with its respective equation that also satisfy the other equation. How could the teacher have used this comment as an opportunity to build the meaning of a system of linear equations in two variables?

  - The teacher could have picked up the idea that each information can be considered separately and each can be represented by a linear equation in two variables with a corresponding table of values all of which satisfy the equation. So this linear equation and its solutions (or the values that satisfy it) can also be considered separately from the other linear equation and its own solutions. But the moments these two equations are considered together or simultaneously, then they comprise a system of linear equations in two variables. So from a discussion similar to the foregoing one, she could have introduced the meaning of a system of linear equations in two variables. Moreover, if their solutions are also considered together or simultaneously, that is the values that satisfy both of them, then this process is known as solving a system of linear equations. So if she had asked either Carol or Christian to specify the ordered pair [which must be (20,5)] that they discovered satisfied both equations, then she could have naturally introduced what the solution of the system means.

OVERCOMING THE BARRIERS TO INTRODUCE LESSON STUDY IN SOUTH AFRICA

Ronél Paulsen

University of South Africa

paulsr@unisa.ac.za

Abstract

In this paper I will be looking at the introduction of Outcomes Based Education in South Africa, and the implications thereof. To understand the implications of the implementation of the concept of Lesson Study in South Africa, I will firstly look at the historical perspective of the process of the development of Education in South Africa. The breaking away from the "old" schooling system was not an easy task. The teachers in South Africa faced a dramatic change from their old practice, since the introduction of Outcomes Based Education placed various demands on their teaching practice, some of which was not understood by the teachers. In order to understand the difficulties teachers are faced with, I will explore various barriers, or rather challenges which will have to be taken into consideration for successful implementation of Lesson Study.

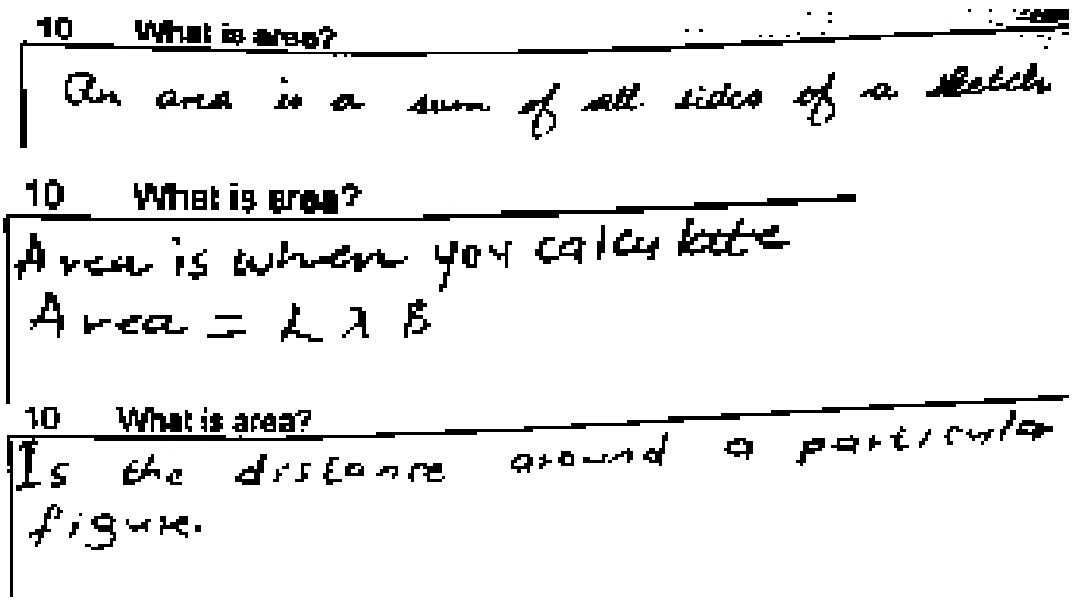
Historical Perspective

During the Apartheid regime in South Africa, each race was classified under its own Department of Education. Teacher training took place at various institutions, and most black teachers in South Africa received training at Colleges where they could either take a two year certificate course, or a three year diploma course at (black) Teachers Training Colleges. No black students were allowed to enrol at so-called "White Universities" This resulted in inadequately trained teachers, and of course, led to inferior teaching at the black schools in South Africa. After the first democratic election in South Africa in 1994, when Nelson Mandela became president of South Africa, a unique constitution changed the lives of all South Africans. In the education sphere, all Colleges of Education closed down, and were incorporated in the twenty-one universities in South Africa. To overcome the legacy of Apartheid, the most significant curriculum reform in SA of the last century was introduced. This was a significant break from the past. The process started off with grave difficulties, and "Curriculum 2005" was revised several times. Curriculum 2005 would be phased in in stages, and by 2009, it should be fully functional in all grades. Curriculum 2005 became synonymous with Outcomes Based Education (OBE). OBE was on the lips of everybody in education, concerned parents, the media and the general public. Few people knew what it was about, and felt threatened by the jargon, which was used to explain the new terms that had to be dealt with. One of the problems was that the advisors from the Department of Education, who were supposed to train teachers, did not understand the notion of OBE themselves, which in turn led to even more confusion. Ultimately the success of the implementation of Outcomes Based Education rests on adequately prepared teachers motivated to teach and support their work. Thus, an enormous task laid ahead for the universities, and the re-training of teachers became a strong focus in the Education Faculties of Universities. In-service training programmes for teachers in South Africa, is an ongoing process.

Content Knowledge

The first and possibly the most important barrier that will have to be overcome is the lack of content knowledge of teachers. Many mathematics teachers do not have a deep enough understanding of the subject matter they are supposed to teach, and do not feel confident of their own understanding. Because of this, teachers are still text-book bound, and traditional teaching methods still prevail. At the advent of OBE, it was advocated that classrooms must become “learner-centred” and that teachers must act as facilitators, in stead of transmitters of knowledge. In theory, this constructivist view of teaching and learning must be applauded, but many teachers misunderstood their role as facilitators, and an “everything goes” attitude was adopted. This created problems when learners who came from the OBE background in Primary Schools, (grade 1 – 7) entered High Schools (grade 8 – 12). Teachers complained that learners had insufficient subject knowledge.

In my involvement in a training programme for teachers in the Intermediate and Senior Phases (grade 4 – 9) I conducted a diagnostic test at the onset of the training programme. Some of the questions and answers are shown here to illustrate the misconceptions teachers had.


10 What is area?

An area is a sum of all sides of a sketch

10 What is area?

Area is when you calculate

Area = L x B

10 What is area?

Is the distance around a particular figure.

I envisage that Lesson Study can play a vital role in the improvement of content knowledge for teachers in South Africa. As Adler (2003 : 5) states, “…teacher education will be more effective if it is focussed on examples of practice and more direct experience in the classroom and alongside experienced teachers”

Resources

The lack of resources is perceived by many teachers as a barrier in their teaching of Mathematics. Most rural schools have a blackboard as their only resource. In the training programme I mentioned earlier, I tried to show teachers that fancy, expensive resources are not always necessary to introduce Mathematical concepts. In the module “Measurement”, I started with the basic concepts and used anything that I could lay my hands on. We used toilet rolls, beans, clay, match sticks and many other manipulative that we could find around the house. For the first time in their teaching careers, these teachers understood the basic concepts of measurements, and were involved in hands-on activities.

In this training session, teachers were engaged in the workshops, but at that stage “Lesson Study” was completely foreign to me, thus, although there were incidents where teachers had to explain their understanding to their peers, we never employed the planning of lessons per se.

The success of Lesson Study in South Africa will also depend on the improvisation of resources. Teachers can become aware of the fact that resources for Mathematics are all around us. However, in over-crowded classrooms in South Africa, where learners work in groups with a set of resources, some of them never touch the manipulatives, and are merely on-lookers of what their peers are doing. Often teachers have to make the resources at their own expense.

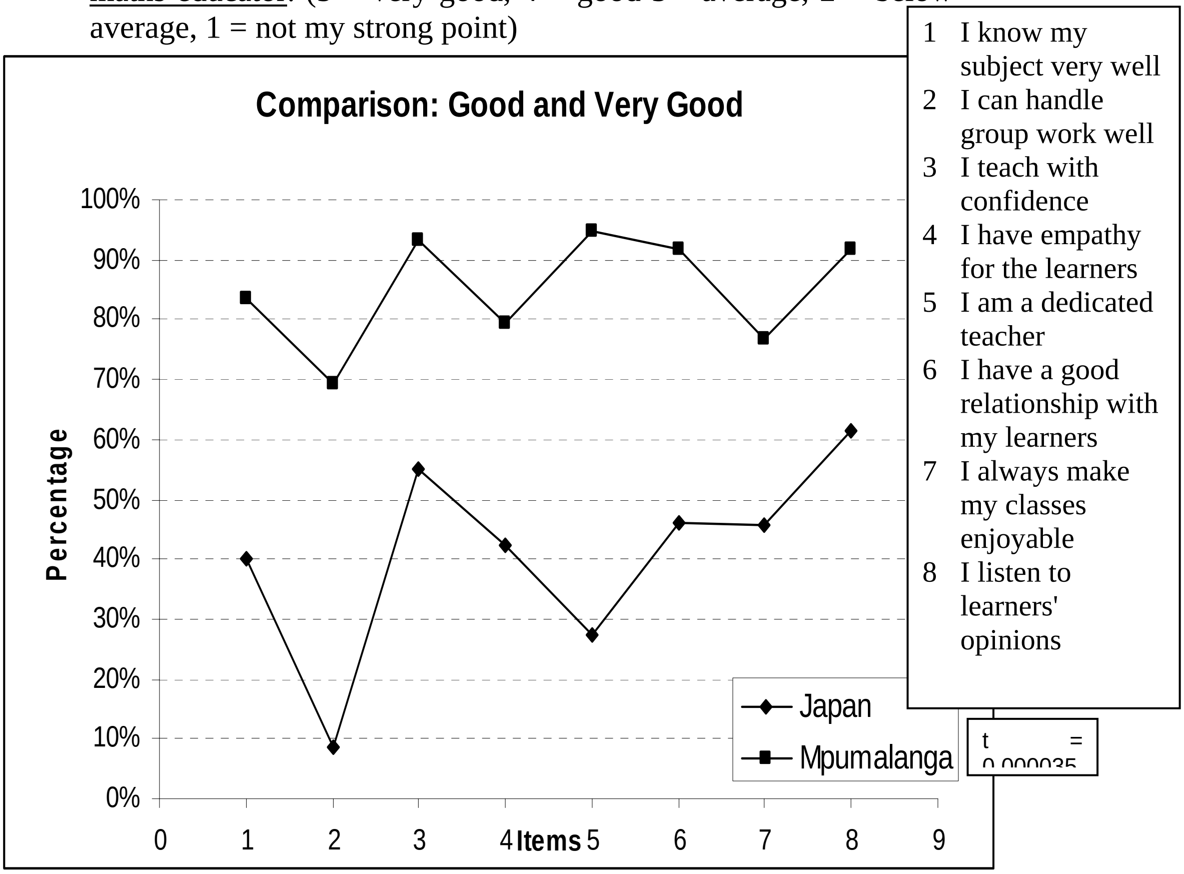
Language Barriers

South Africa has eleven official languages. This includes nine African languages, English and Afrikaans. Although the official language of teaching is either English or Afrikaans, we find that in the rural areas, especially in Primary Schools, that the language mostly spoken in schools is that of a particular community. In urban schools, multilingual classrooms where learners of any of the eleven languages could be in the same classroom, but English is the dominant language. English is sometimes the second or even third language for some learners, therefore teachers use code-switching as a pedagogical strategy. Code switching occurs when the teacher or learners switch from one language to another. Teachers are therefore faced with the major challenge with continuously teaching Mathematics, but also English at the same time. Multilingual classrooms indeed place a far more profound demand on teachers in South Africa, than in first world countries.

Teachers' Own Perceptions of Their Classroom Practice

In my survey of classroom practices between teachers in Japan and Mpumalanga (a province in South Africa), a very interesting phenomena was observed. The following question was posed:

Please give yourself a rating for each of the following quality as a maths educator. (5 = very good, 4 = good 3 = average, 2 = below average, 1 = not my strong point)



The graph above speaks for itself. In view of the successes of Japanese students in TIMSS, it seems as if the teachers from Mpumalanga exhibited an inflated perception of their subject knowledge. I do not think it is intrinsically bad to have a positive perception of your own classroom practices, but when teachers in South Africa are exposed to Lesson Study, I am sure they will benefit from the consequences of sharing which lies at the heart of Lesson Study.

Teachers afraid of "intruders" in their classrooms

At a recent conference of Independent Schools in Pretoria, I became aware of the fear teachers have to allow "strangers" in their classrooms. Most teachers showed no interest in becoming part of a Lesson Study group. The challenge to me will thus be to start on a small scale, and use platforms such as the annual AMESA (Association of Mathematics Educators in South Africa) conference which will be held in July this year, to advocate the advantages of this practice of in-service training and professional development. Teachers will firstly have to be convinced that Lesson Study must not be seen as invasion of their classrooms. They should be made to feel confident that Lesson Study is only a tool that has enormous implications for the improvement of, not only their teaching, but also for the learning that takes place in a classroom. Only when this barrier is overcome, and teachers do not feel threatened by this "new" way of in-service training, can there be the slightest of beginnings with this endeavour.

Conclusion

As pointed out, there are several barriers that need to be overcome before Lesson Study can be implemented successfully in South Africa. These, however are rather seen as challenges. A small scale work on Lesson Study will be undertaken in Pretoria, and once this is established, a wider circle of schools will be included. This project must be seen as a long term endeavour, and the ultimate success thereof will depend on the impact it has on the preliminary accomplishments.

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USING LESSON STUDY AS A MEANS TO INNOVATION FOR TEACHING AND LEARNING MATHEMATICS IN VIETNAM RESEARCH LESSON ON THE PROPERTY OF THE THREE MEDIANS IN A TRIANGLE

Tran Vui
College of Education, Hue University, Vietnam

Introduction

In Vietnam, the reform mathematics curriculum requires more than mastery of basic mathematical skills, good algorithms in solving a class of specific problems. The teaching of mathematics is changing. We are seeking for the innovation of teaching and learning mathematics. The teacher ought to think of teaching in terms of several principal hands-on activities, problematic real life situations, and open-ended questions. The innovation of teaching is to help students construct their own knowledge in an active way; enhance their thinking through solving non-routine problems while working cooperatively with classmates so that their talents and competencies are developed. There are several possibilities for innovation of mathematics education in an economy. Lesson study which originated from Japan is currently a central focus in US and other economies for the professional development of teachers and the improvement of students' learning.

In this research paper on lesson study for developing good lesson, we adopted a lesson study cycle comprising planning implementing and observing discussing and reflecting in our economy. The research focuses on lesson study as a means to innovation. The results from this lesson study showed that good teaching practices are powerful models for changing the quality of mathematics education. We developed a VTR of good lesson as a product of our lesson study and to use it for teacher education.

1. Planning

Since the Vietnamese secondary mathematics teachers who involved with this research were not familiar with the use of lesson study to improve their good practices in their classrooms. So first we had to conduct a workshop on "Lesson study as a means to innovation of teaching and learning mathematics". Twelve teachers, one specialist in mathematics attended this workshop; they were from the lower secondary school Nguyen Tri Phuong, Hue City, Vietnam. The objectives of the workshop were:

- to help teachers on how to use lesson study as a means to innovation of teaching and learning mathematics;
- to help teachers on how to use the innovation to improve students learning;
- to discuss with teachers on how the lesson study support the professional development of teachers;
- to help teachers on how to use innovation in teaching and learning mathematics to implement the reform mathematics curriculum;
- to select a well known and experienced teacher to prepare the lesson plan and carry out it in the class for observing and discussing.

At the end of the workshop, a group of teachers was formed to be involved in this study. This first team worked as a research group to create the lesson plan, worksheets and instructional materials suitable to the unit selected from the reform mathematics curriculum for grade 7 in Vietnam.

The unit “*The property of the three medians in a triangle*” chosen by teachers located in the text book, page 65-66, Volume 2, 2003. The teachers agreed in the meeting that the content of this unit is difficult and abstract to the students. The presentation of this unit in the text book is not meaningful. Traditionally, the students have to accept the definition of the median from the text book. The definition: “*The segment AM joining the vertex A of the triangle ABC with the midpoint M of BC is called the median of the triangle ABC*” is stated directly. In this research, teachers created problematic situations to help students explore the concept of medians and their properties meaningfully.

The study aimed to explore and investigate the implementation of lesson study as a means to innovation of teaching and learning of selected topics in lower secondary mathematics in Vietnam.

The research sought to find answers to the following questions:

1. How does the lesson study as a means to innovation affect to teaching and learning mathematics?
2. How does the innovation affect to the improvement of students learning?
3. How does the lesson study support the professional development of teachers?
4. How does the use of innovation in teaching and learning mathematics affect to the implementation of reform curriculum?

Findings of the study will shed light on the relative contribution of the lesson study as a means to innovation of teaching and learning mathematics.

The study was conducted in two months March - April 2006. All teachers were introduced to lesson study for the first time at the workshop of the research. Also at the workshop the methodology of the research was explained and discussed, i.e. that the teachers were responsible for their own use of innovation in teaching mathematics. What were required of them were observations on the things which happened in their classes and their reactions to the innovation.

Three classes were involved in the study. The students’ ages ranged from 12-13 years. Overall a total of 145 students and 8 teachers were involved in the study. The study involved grade 7 students. The topic covered in the grade 7 was the property of the three medians in a triangle.

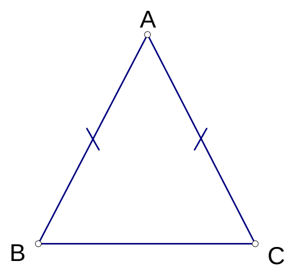
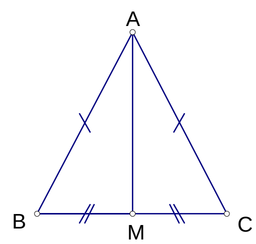
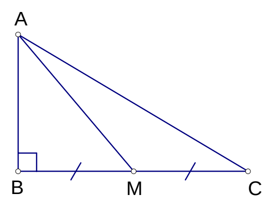
To prepare the lesson plan, we considered the role of this unit in the curriculum and discussed what teachers usually taught this unit. Teachers agreed that the lesson plan should have some characteristics as follows:

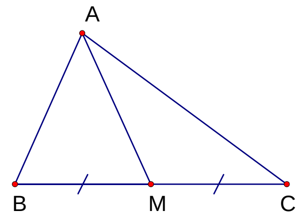
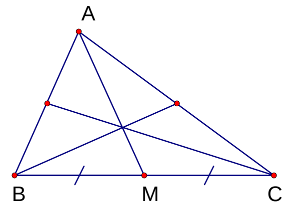
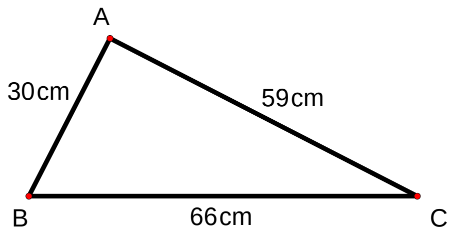
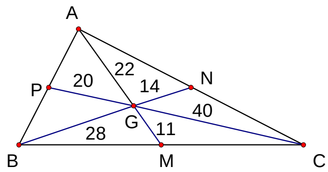
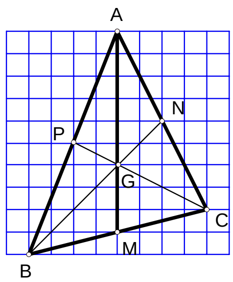
- The mathematics content taught is meaningful;
- The thinking processes of students are transparent through their answers, products, presentation that the viewers can recognize while watching the video.
- The innovation in teaching and learning is discussed, prepared in the mathematics division of the school. Every teacher in the division has his/her own contribution to the innovation.
- The lesson uses the instructional materials that are innovative and appropriate to the school.

The teacher implements this lesson plan will be chosen by teachers in the division. He has experienced in creating problematic situations and asking open-ended questions that require mathematical thinking of students.

2. Implementing and observing

We implemented the lesson plan designed by Mr. Nguyen Khoa Tu in two different classes before shutting video. In the first class, there were seven observers; they were mathematics teachers of the school. Through discussion, teachers found out that the lesson plan needs to be changed at some points to help students answer the open-ended questions in problematic situation. Some questions was not clear and general so students felt not confident to answer. Some questions required only remember and recall facts and students were not interested in answering the questions. So we revised the lesson plan and taught in another class. In the second class teachers observed students folding, drawing and measuring and found out that students had good responses to questions and actively engaged to the tasks. This time, the teachers agreed that the lesson plan and its lesson flow were suitable to our students at every class grade 7. But the lesson still has something need to be renewed. Then we decided to implement the revised lesson plan the third time in an actual class for shutting video. The students responses to instructional activities at consideration points and evaluation were illustrated in the following table.

Instructional activities	**Students responses to consideration points**
Activity 1 1. How to divide the triangular cake into two equal parts?	**S**: What do you mean by two equal parts? Equal in shape? **S**: I think, their areas are equal, because the amount of cake of each part is the same. **S**: Can we cut the cake by a straight line?
2. Divide an isosceles into two equal parts?  	**S**: It looks easier. I draw the segment from *A* to midpoint *M* of *BC*. And cut through segment *AM*. **S**: We have two triangle *MAB* and *MAC* equal (side – angle – side), so *AM* is the height. Then S(*MAB*) = S(*MAC*) = $\frac{1}{2}AM\times\frac{BC}{2}$. 
3. Divide a right triangle into two equal parts?	**S**: Construct midpoint *M* of *BC*. Draw segment *AM*. Then *AB* is the height of two triangles *MAB* and *MAC*. 

4. General case: Divide an arbitrary triangle two equal parts? *AM* is called the median of triangle *ABC*.	**S**: Now I can see the way to divide the cake. Draw midpoint *M* of *BC*. *AM* will divide the triangle into two equal parts. 
Activity 2 **Q1**. How many medians can you draw in a triangle? Why? **Q2**. What do you mention about the three medians in your figure?	**S**: We always can draw three medians from three vertices. **S**: Three medians are convergent at one point. 
Activity 3. **Task1**. Students are given a triangle drawn on A2 paper without grid.  **Q**. Can you find any ratio of the lengths of segments that determined by the medians.	**S**: We fold *B* to *C* to get midpoint *M*. And then *N* and *P*. **S**: Use ruler to measure the lengths of segments.  *GM* = 11cm; *AG* = 22cm. *GN* = 14cm; *BG* = 28cm. *GP* = 20cm; *CG* = 40cm. **S**: I think *AG* = 2*GM*; *BG* = 2*GN*; *CG* = 2*GP*.
Task 2. **Q**. Can you find any ratio of the lengths of segments that determined by the medians.	**S**: From the grid we can define the midpoints *M*, *N*, *P*. They are the centers of corresponding rectangles. **S**: I see that *AG* = 2*GM*; *BG* = 2*GN*; *CG* = 2*GP*. But I do not know how to prove it. 
Task 3. Open Geometer's Sketchpad, draw a triangle and its three medians. Apply **Measure** | **Length** to measure lengths. Apply **Measure** | **Calculate** to calculate ratio.	**S:** Use GSP to draw a triangle and its medians. Measure lengths and calculate ratio. **S:** Drag point A to change the triangle. Observe the behavior of the ratio. **S**: *AG* = *2GM*; *BG* = *2GN*. When this group presented their work to the whole class on computer, most of students were surprised because the vertices *A*, *B*, *C* can be dragged but the ratios unchanged.

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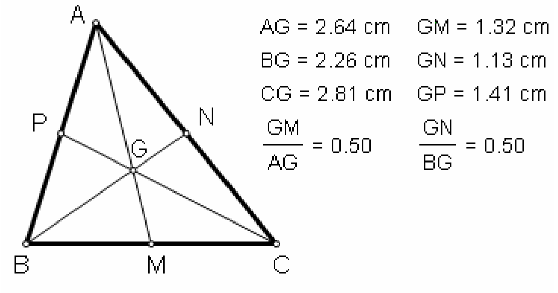
<td>
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<tr>
<td>Exercise 1
Exercise 2
These two exercises aim to consolidate what the students have learnt. The questions are presented on LCD by PowerPoint presentation. (see Lesson Plan in Appendix).</td>
<td>Most of students called upon by teacher answered the questions quite fast. Students showed their understanding and can apply the theory to answer some exercises.</td>
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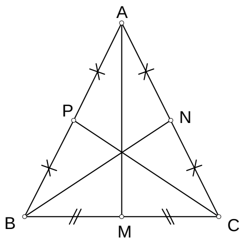
<td>Problem 1.
Given an isosceles ABC. What is about its three medians? What happens in the special case of an equilateral?

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<td>S: An isosceles is symmetry. So I think BN = CP.
S: We need to prove PBC = NCB.
S: BC common, BP = CN, PBC = NCB. So PBC = NCB. Thus BN = CP.
S: An equilateral is a special isosceles, so three medians are equal.</td>
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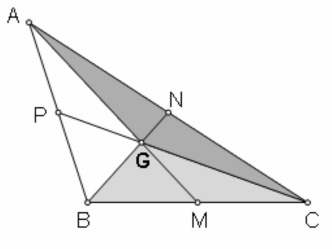
<tr>
<td>Problem 2.
Given a triangle ABC. Divide the triangle into 3 equal parts?
What is about six equal parts?

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<td>S: I see that the areas of six small triangles are equal.
S: S(MAB) = S(MAC), and S(MGB) = S(MGC), so taking away the same areas we have S(AGB) = S(AGC).
S: Similarly S(AGC) = S(GBC)</td>
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</table>

2. Discussing and reflecting

There were thirteen mathematics teachers observed the class including Mr. Tran Du Sinh, Mr. Nguyen Dinh Son (mathematics specialists, Department of Education and Training, Thua Thien Hue), Dr. Tran Vui, Mr. Le Van Liem, Mr. Tran Kiem Minh (Department of Mathematics, Hue University), Mr. Nguyen Huu Bi (The principal), classroom teachers: Mr. Dinh Van Luong (Head of Division), Mr. Nguyen Van Thang, Mr. Tran Van Dien, Mrs. Cao Thi Kim Nhung, Mr. Le Van Cam, Mrs. Tran Thi Thang, Mrs. Nguyen Thi Xuan. After observing the actual class instructed by Mr. Nguyen Khoa Tu, we organized a meeting for sharing ideas and comments. We discussed the following issues. In the meeting, the teachers gave a lot of comments to four main issues with corresponding questions that were recorded as follows.

1. **Lesson study as a means to innovation**

In lesson study teachers played a central role to decide what the innovation in teaching and learning is. They are the persons to implement the innovations in their actual classrooms. Teachers help teachers to improve mathematics instruction in the classroom. The innovation can be shared to other teachers.

What was the innovation in teaching method that appeared in the lesson?

We got many answers to this question:

- Lesson started with a real life situation by asking students divide a real cake. The learning process involves with all students working in small groups.
- Students actively sought for and explored mathematical knowledge with the help of teacher.
- Teacher used the way of posing a problem that had the root from real-life situation to make students getting interest at the starting point of the lesson.
- The lesson was student centered, cooperative learning. From a problematic real life situation, teacher facilitated students seek for and construct new knowledge.
- Students actively worked with mathematical problems.
- The lesson is innovative; it is different with old approach of teaching by lecturing.

2. **The improvement of students learning**

We are seeking for the good practice to improve students learning. Good practice embodied in this lesson study is based on outcomes of successful students learning, including students mathematical thinking, and can be used for further development or challenges.

Was the mathematical content taught in the lesson meaningful and realistic?

- Students understand the relationship between mathematics and real life.
- By folding papers, measuring lengths on papers, students explored the property of three medians. Students gave good comments on the medians of isosceles and equilateral.
- The lesson started from a real situation to develop meaningful mathematics knowledge and then students applied constructed knowledge back to the real life problem.
- Mathematical concept was constructed from a familiar situation of the real world. The knowledge constructed in the lesson helps students solve real life problems.
- Students felt that really have a linkage between mathematics and real life.

How did the key points that intend to enhance students' mathematical thinking show in the lesson?

- Students showed good responses to the questions, but it depends on the ability of each class to have appropriate questions.
- Practicing measurements, inducting from concrete data to generalize the mathematics property.
- The open ended questions gave students chance to explore the property of three medians by themselves.
- Students had good comments on the medians of an isosceles.
- Students understood the way two divide a triangle into three equal parts.

How did the ability of students in responding the questions and tasks requiring mathematical thinking of students show in the lesson?

- The tasks and questions were relevant to students' previous knowledge so they feel confident to seek for new knowledge.
- Most of students apply mathematical reasoning to explain new knowledge they have found.
- Students stated the results explored by themselves accurately.

3. Lesson study supports the professional development

How did the instructional materials support the lesson?

- The instructional materials helped the lesson a lot. They supported students explore and find out new knowledge.
- Low cost instructional materials such as paper, grid paper combined with modern computer helped students explore successfully mathematical ideas.
- Teacher used many kinds of instructional materials that helped students explore corresponding mathematical ideas effectively.
- We need to have an in-service training course for developing instructional materials, especially computer software.

How did the interaction student – student – teacher show in the communication and discussion?

- The students worked in small groups with the guidance, evaluation of teacher. The hint of teacher was effective in discussing between students and students.
- Some students were hesitated and shy to share their knowledge with friends.

What should be changed in the lesson to improve the learning study next time?

- One student should have a separate triangle on paper, so he can fold the paper to explore the property of three medians. After exploring, students discuss in groups to explore the property of centroid.
- This lesson can apply broadly to other classes, but we need to improve the professional ability of teachers and reform the students' assessment.

4. Innovation to the implementation of reform curriculum

How did the thinking process of students show in doing specific mathematical tasks in the lesson that were identified in the reform curriculum?

- Students explore exactly mathematical property of the three medians by observing, folding, measuring and inducing.
- Students can apply what they have learnt to solve some specific problems posed by teacher.
- Most of students showed that they understood the lesson, solved the problems set by the teacher. These problems were revised from the text books.

What is about the application of this lesson plan in the curriculum of lower secondary mathematics?

- With some schools having good facilities such as computer, LCD this lesson plan will be very effective.
- We need to apply this lesson study to other topics and other classes.

- The curriculum is still heavy, a lot of content knowledge in the text book that teachers have to deliver, so the time constraint is a big issue for all students to do the mathematical work by their own paces.
- We need to have relevant facilities in school to prepare appropriate instructional materials for specific topics in the curriculum.
- We need the practical theories that help classroom teachers develop innovation that relevant with the curriculum.

Conclusions

This is the first time we introduced the lesson study cycle in a school. All mathematics teachers in the school agreed that lesson study provides them a good opportunity to see teaching and learning in the classroom scenarios. From that actual scenario teachers develop innovative teaching practices to help students learning. The use of innovation to teaching and learning mathematics in the classroom must be implemented to engage students in meaningful mathematical tasks that require higher order thinking. The innovation provides all students access to a broad range of mathematical ideas. Specifically, the research sought to find answers to the research questions:

1. Lesson study guides teachers to focus their discussions on getting the effective innovation through the cycle. By discussing and sharing new ideas on innovation, observing what happens in actual classroom, teacher improves their teaching and enhances the students learning. We can apply lesson study to many topics in the curriculum. Lesson study as a means to innovation actively affected to teaching and learning mathematics in the school.
2. The innovation as a product of the lesson study helps students have better and meaningful understanding of difficult mathematical concepts. Students were able to discuss and interact freely with their pairs/groups while answering open-ended questions relevant to them. The students communicated friendly their mathematical thinking while they are engaged in the mathematics activities. With hand-on activities, students always have something to share with their friends about problems involving with mathematical thinking.
3. The lesson study for good practice in teaching and learning mathematics actually supported the professional development of teachers. Teachers learnt some things new from their peers and can apply them to the teaching mathematics.
4. The reform mathematics curriculum requires students learning mathematics in an active way to enhance mathematical thinking, so the innovation in teaching and learning mathematics can help teachers implement effectively the curriculum.

Acknowledgement. This lesson study was conducted in Hue City, Vietnam under the collaborative framework involving mathematics education among the APEC Member Economies. Special thanks due to the principal, mathematics teachers of the lower secondary school Nguyen Tri Phuong, Hue City, Vietnam for their contribution to the research.

Reference

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2. Akihiko Takahashi and Makoto Yoshida (2006). *Developing good mathematics teaching practice through lesson study: A U. S. perspective.* Paper presented at APEC-Tsukuba International Conference, January 2006, Tokyo, Japan.
3. Catherine Lewis (2006). *Professional development through lesson study: Progress and Challenges in the U.S*. Paper presented at APEC-Tsukuba International Conference, January 2006, Tokyo, Japan.
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Appendix

LESSON PLAN

Mathematics Lesson Grade 7
Teacher: Mr. Nguyen Khoa Tu, Senior Teacher, Nguyen Tri Phuong Lower Secondary School, Hue City, Vietnam.
Students' ages: 12 years old.
Research Theme: Examining instruction that will help students have the relations between their own experience in dividing a triangular cake into two equal parts and the median, explore the property of three medians and the corresponding ratios by practicing and answering open-ended questions.
Section: 53 in Vietnamese mathematics grade 7 textbook (45 minutes). Topic: *The properties of three medians in a triangle*.

1. Objectives

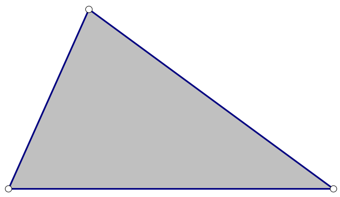
- Students grasp the concept of the medians, centroid of a triangle.
- Understand the convergence of three medians, the property of the centroid through practical works, measuring, drawing and folding papers.
- Know how to draw a median of a triangle, gain skills in using properties of triangle to solve some simple exercises, problems.
- Through the lesson, the teacher creates problematic situations and poses the open-ended questions to enhance students' critical and creative thinking.

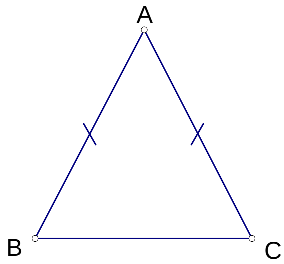
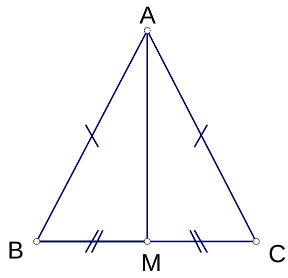
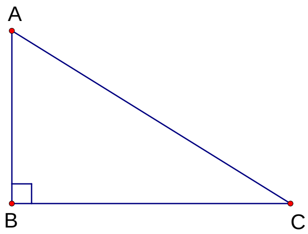
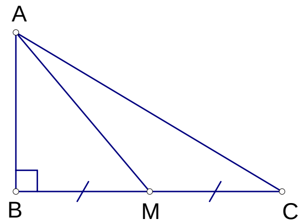
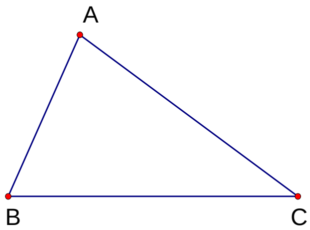
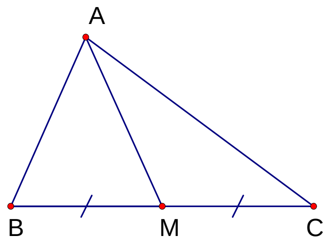
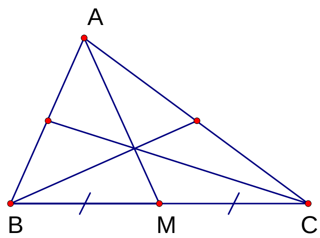
2. Preparation

Students: Rulers, compasses, pencils, transparency papers.
Teachers:

- 2 different triangles drawn on A2 paper.
- 3 different triangles drawn on grid A2 paper.
- One triangle drawn in GSP software.
- The PowerPoint file of the lesson plan, LCD.
- Overhead projector and transparency papers.

3. Flow of the lesson

Content	*Instructional Activities*	*Points of Consideration & Evaluation*
1. **Introduction Activity** Getting students familiar with the new concept of the median.	**Activity 1**. - Teacher gives students a real triangular cake. Teacher asks students how to divide the cake into two equal parts.  - Teacher asks students to start with two special cases on the board.	Students show their own experience on two equal parts and the area of a triangle. Their areas are equal.
	Isosceles:	S: Draw the segment from *A* to midpoint *M* of *BC*.

		 Show that: S(*MAB*) = S(*MAC*)
	Right triangle: 	Similarly, 
	General case: Arbitrary triangle 	Generally, 
2. **Understand** concept of the median, and the procedure to draw a median of a triangle.	The segment *AM* is called the median of the triangle *ABC*. The procedure to draw a median of a triangle. **Activity 2**. **Q1**. How many medians can you draw in a triangle? Why? **Q2**. What do you mention about the three medians in your figure?	Each student draws a triangle on a piece of paper. Then draw three medians.  The three medians are convergent at one point.
3. **Explore** the property of three medians	**Activity 3**. Students are divided into 6 small groups. Two groups have the same task. Each group works on one task. Which group has good answer will present to the whole class for discussion. **Task1**. Students are given a triangle drawn on A2 paper without grid.	By having students engage in folding paper, drawing, measuring three medians. Three medians are convergent at point *G*.

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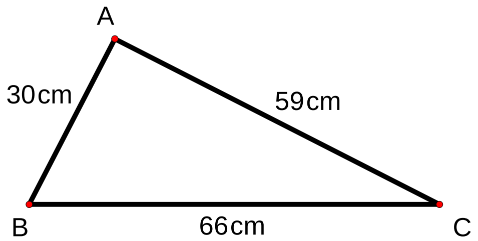

By folding the edges, determine the midpoint of each leg. Draw three medians of the triangle, measure their lengths.
Q1. What is the relation between three medians?
Q2. Can you find any ratio of the lengths of segments that determined by the medians?
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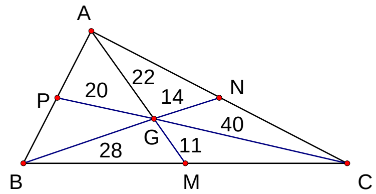

Using ruler to measure:
GM = 11cm; AG = 22cm.
GN = 14cm; BG = 28cm.
GP = 20cm; CG = 40cm.
Conclusion:
$AG = 2GM$
$BG = 2GN$
$CG = 2GP$
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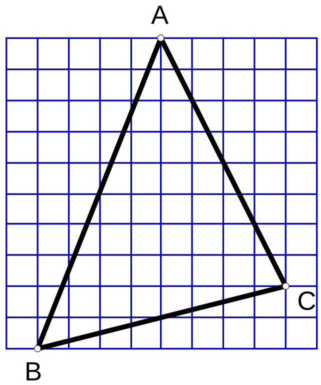
Task 2. Students are given a triangle drawn on A2 grid paper. Grid 5cm×5cm. Determine the midpoint of each leg. Draw three medians of the triangle, identify their lengths.



Q1. What is the relation of three medians?
Q2. Can you find any ratio of the lengths of segments that determined by the medians?
Theorem:
In a triangle three medians are convergent at centroid, and the length from centroid to a vertex is $\frac{2}{3}$ of the median passing through that vertex.
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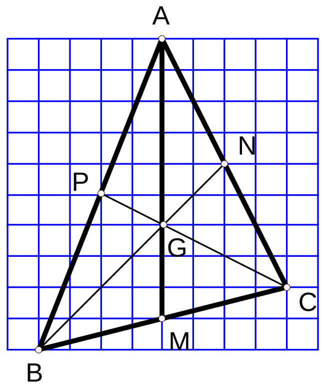
By having students engage in determining the midpoint of each edge, drawing three medians and identifying the lengths of segments that determined by the medians. Three medians are convergent at point G.



$AG = 2GM$
$BG = 2GN$
$CG = 2GP$
</td>
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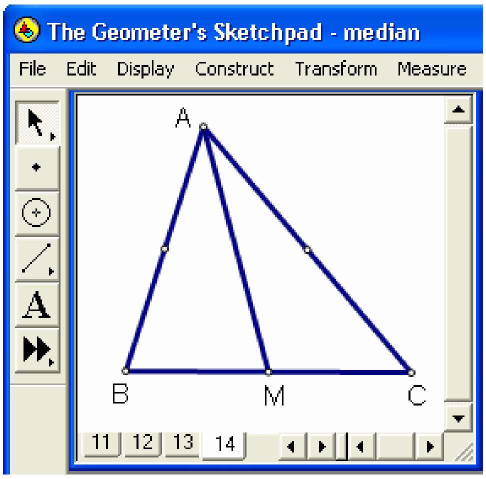
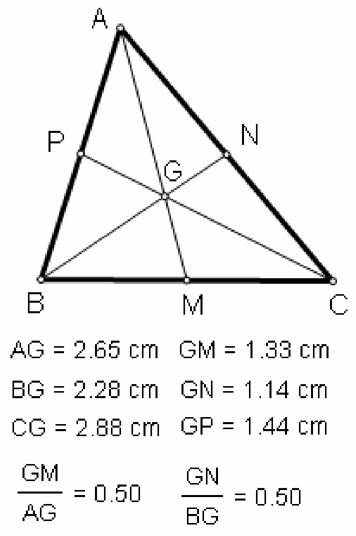
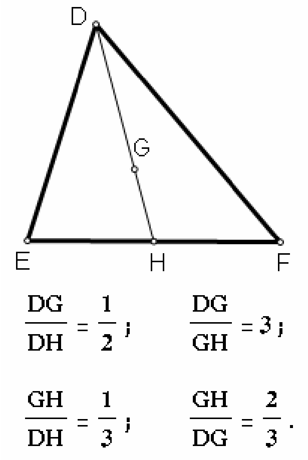
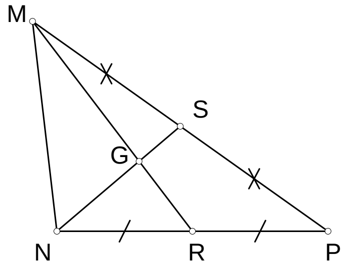
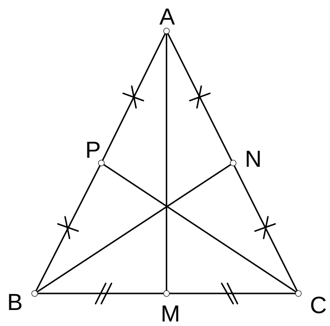
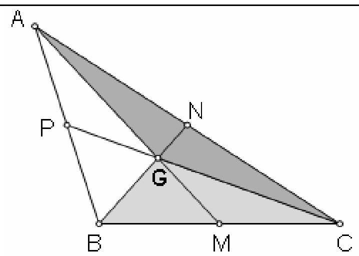
<td>
Task 3. This task will be used only in the class that has computer and LCD projector.
Open Geometer's Sketchpad; draw a triangle and its three medians. Apply
</td>
<td>
Students use GSP to draw a triangle and its medians. Measure lengths and calculate ratio.
Drag point A to change the
</td>
</tr>
</table>

	Measure | **Length** to measure lengths. Apply **Measure** | **Calculate** to calculate ratio.  **Q1**. What is the relation of three medians? **Q2**. Can you find any ratio of the lengths of segments that determined by the medians?	triangle. Observe the behavior of the ratios. Conclusion. 
4. **Consolidate** the theorem.	**Exercise 1** Let G be the centroid of triangle DEF with the median DH. In the following statements which is correct?  	Students apply what they have explore to choose the correct statement: $\frac{DG}{DH}=\frac{2}{3}$; $\frac{DG}{GH}=2$; $\frac{GH}{DH}=\frac{1}{3}$; $\frac{GH}{DG}=\frac{2}{3}$.
	Exercise 2 Given the figure below. Fill in the blanks to have correct equations.	Students recognize different ratios can be gain from the medians and centroid. $MG = \frac{2}{3}MR$; $GR = \frac{1}{3}MR$; $GR = \frac{1}{2}MG$;

	 a. *MG* = . . . *MR*; *GR* = . . . *MR*; *GR* = . . . *MG* b. *NS* = . . . *NG*; *NS* = . . . *GS*; *NG* = . . . *GS*.	$NS = \frac{3}{2} NG$; *NS* = 3*GS*; *NG* = 2*GS*.
6. **Problem solving**	**Problem 1** Given an isosceles *ABC*. What is about its three medians? What happens in the special case of an equilateral? 	By having students engage in reasoning, making conjecture that *BN* = *CP*. And then prove it. Consider two triangles *PBC* and *NCB*. *BC* common, *BP* = *CN*, *PBC* = *NCB*. So *PBC* = *NCB*. Thus *BN* = *CP*. In an equilateral, three medians are equal.
	Problem 2 Given a triangle ABC. Divide the triangle into 3 equal parts? What is about six equal parts?	 Since S(*MAB*) = S(*MAC*), and S(*MGB*) = S(*MGC*), thus S(*AGB*) = S(*AGC*). Similarly S(*AGC*) = S(*GBC*)

APPENDIX: RELATED DOCUMENTS

List of Participants from Economies/ Countries

NAME	ECONOMY/COUNTRY	AFFILIATION
Alan Bishop	Australia	Monash University
Doug Clarke	Australia	Australian Catholic University
Babara Clarke	Australia	Monash University
Max Stephens	Australia	University of Melbourne
Wee Toing Seah	Australia	Monash University
Grecia Ga'lvez	Chile	Ministerio of Education
Wang Wei	China	Plan China Churhua PU
Qi Tao	China	Plan China Churhua PU
Shizumi Shimizu	Japan	University of Tsukuba
Prof. Masami Isoda	Japan	University of Tsukuba
Kazuyoshi Okubo	Japan	Hokkaido University of Education
Fumi Ginshima	Japan	Curriculum Research Center National Institute for Educational Policy Research
Hiroko Tsuji	Japan	Hokkaido University of Education
Takeshi Miyakawa	Japan	University of Tsukuba
Hiroshi Shimahara	Japan	67-46 IBUKINO MIDORY Yokohoma, Japan
Takehiko Ohtani	Japan	2-17-1 Horifure Kitc-ku Tokyo Japan
Frederick Leung	Hong Kong	University of Hong Kong
Marsigit	Indonesia	FRIDA State University of Yogyakarta
Sukirman	Indonesia	FRIDA State University of Yogyakarta
Kham Ane Sayasone	Laos PDR	National University of Laos
Xaya Chemcheng	Laos PDR	National University of Laos
Lim Chap Sam	Malaysia	University of Science Malaysia
Lee Siew Eng	Malaysia	University of Malaya
Kor Liew Kee	Malaysia	UiTM Cawangan Kedah 08400 Merbok Kedah.
Fatimah Saleh	Malaysia	Universiti Sains Malaysia
Soledad A Ulep	Philiphines	University of the Philippines
Yeap Ban Har	Singapore	Nanyang Technological University
Christine Kim-Eng LEE	Singapore	National Institute of Education
Fang Yanping	Singapore	2-02-24 NIE / Nanyang Walk. Sinyapore 6
Ho Siew Yin	Singapore	National Institute of Education
Jeanne Wolf	Singapore	National Institute of Education
Ariken Charmaine Carrie	Singapore	Cedar Primary School
Chua Ee Ling Isabelle Elaine	Singapore	Cedar Primary School
NG Luan Eng	Singapore	National Institute of Education

NAME	ECONOMY/COUNTRY	AFFILIATION
Sean Teng Ta Shen	Singapore	Cedar Primary School
TEO Soh Wah	Singapore	National Institute of Education
Yap Lai Boon	Singapore	Cedar Primany School
Yeo Shu Mei	Singapore	National Institute of Education
Yeo Teong Hiang	Singapore	Cedar Primary School
Ronel Paulsen	South Africa	University of South Africa
Akihiko Takahashi	USA	DePaul University
Libby Knott	USA	The University of Montana
James R. Kennis	USA	Columbia University
Tran Vui	Vietnam	College of Education, Hue University

List of Participants from Thailand

NAME	AFFILIATION
Sunthorn Chaichana	Sakonnakhon Rajabhat University
Anchalee Tananone	ChiangMai University
Tipparat Noparit	ChiangMai University
Praewpisut Chantate	Pathumwan Institute of Technology
Tipaval Phatthanangkul	Thammasat University
Luddawan Pensupha	Thammasat University
Ardoon Jongrak	Phetchabun Rajabhat University
Chainarong Khunpanuk	Phetchabun Rajabhat University
Rachada Chaovasetthakul	Print of Songkla University
Prapasri Assawakul	Suranaree University of Technology
Eckart Schulz	Suranaree University of Technology
Chanon Chuntra	Kasetsat University
Mongkol Vongpayak	Kasetsat University
Supotch Chaiyasang	Srinakarinwirot University
Rungfa Janjaruporn	Srinakarinwirot University
Charinee Triwaranyu	Chulalongkorn University
Supasiri Senarith	Office of Knowledge Management and Development
Apiradee Kaotain	Office of Knowledge Management and Development
Vorapoj Prasanpanich	Office of Knowledge Management and Development
Wipas Sarutpong	Office of Knowledge Management and Development
Somsong Suwapanich	Rajabhat Maha Sarakham University
Narong Saratassananan	Loei Rajabhat University
Arpornrat Saratassananan	Loei Rajabhat University
Uthumporn Palavong	King Mongkut's Institute of Technology North Bangkok
Wichai Chumni	Thaksin University
Utith Inprasit	Ubonrajathani University
Kiet Sangaroon	Khon Kaen University

NAME	AFFILIATION
Waiyawut Intavong	Nakhon Si Thammarat Rajabhat University
Sakorn Bundao	Sukhothai Thammathirat OpenUniversity
Suwattana Eamoraphan	Chulalongkorn University
Ganchana Sucheenapong	Valaya Alongkorn Rajabhat University
Charuwan Singmoang	Rajabhat Rajanagarindra University
Pornsin Supawan	Rajabhat Rajanagarindra University
Ronnachai Panapoi	The Institute for the promotion of Teaching Science and Technology
Yuangrat Wedel	Plan Organization, Thailand
Suchada Khreersinhul	Plan Organization, Thailand
Phiranant Numkanisorn	Assumption College Nakhonratchasima
Siriphat Pukpo	Northeastern Bilingual School
John Auseth	Northeastern Bilingual School
Suladda Loipha	Khon Kaen University
Maitree Inprasitha	Khon Kaen University
Nit Bungamongkon	Khon Kaen University
Paisarn Suwannoi	Khon Kaen University
Pennee Narot	Khon Kaen University
Ladda Silanoi	Khon Kaen University
Auijit Pattanajak	Khon Kaen University
Chantana Klomjit	Khon Kaen University
Channarong Heingraj	Khon Kaen University
Kingfa Sintoovongse	Khon Kaen University
Terachai Nettanomsak	Khon Kaen University
Narumol Inprasitha	Khon Kaen University
Neon Pinpradit	Khon Kaen University
Pasaad Kongtaln	Khon Kaen University
Wallapha Areeratana	Khon Kaen University
Sarintip Raksasataya	Khon Kaen University
Noytip Limyingcharoen	Khon Kaen University
Parichart Vachiruddanupap	Khon Kaen University
Wimol Sumranwanich	Khon Kaen University
Sumalee Chaijaroen	Khon Kaen University
Chokchai Yeunyong	Khon Kaen University
Varisara Srisawat	Demonstration School, Khon Kaen University
Petcharat Chongnimitsataporn	Demonstration School, Khon Kaen University
Sopa Chunharatchaphan	Demonstration School, Khon Kaen University
Chujit Yuangsaard	Demonstration School, Khon Kaen University
Napaporn Voranetsudatip	Demonstration School, Khon Kaen University
Kanyarat Samrit	Demonstration School, Khon Kaen University
Amphaporn Boriwong	Demonstration School, Khon Kaen University
Sukhuma Akaram	Demonstration School, Khon Kaen University
Supalaksana Aunpromma	Demonstration School, Khon Kaen University
Sunantha Leeprakorbboon	Demonstration School, Khon Kaen University
Pensri Sangaroon	Demonstration School, Khon Kaen University
Metta Mawiang	Demonstration School, Khon Kaen University
Suphanat Siri-Aisoon Klinhom	Demonstration School, Khon Kaen University

NAME	AFFILIATION
Chatchai Prarat	Demonstration School, Khon Kaen University
Kuajit Chimtim	Demonstration School, Khon Kaen University
Passara Intarakumhaeng	Demonstration School, Khon Kaen University
Nittaya Chotikarn	Demonstration School, Khon Kaen University
Worathep Chimtim	Demonstration School, Khon Kaen University
Mongkhol Prasertsang	Demonstration School, Khon Kaen University
Wichai Wiangwisad	Demonstration School, Khon Kaen University
Sungwean Pinagalung	Demonstration School, Khon Kaen University
Sudjai Seeja-morn	Demonstration School, Khon Kaen University
Supaporn Prasarnphanich	Demonstration School, Khon Kaen University
Aree Phawattana	Demonstration School, Khon Kaen University
Rattana Atanawong	Demonstration School, Khon Kaen University
Udomporn Supannavong	Demonstration School, Khon Kaen University
Smitra Sornsuk	Demonstration School, Khon Kaen University
Parichart Kaennakum	Demonstration School, Khon Kaen University
Jirakan Hongchuta	Demonstration School, Khon Kaen University
Sopa Khumseethum	Demonstration School, Khon Kaen University
Samruan Chinjunthuk	Demonstration School, Khon Kaen University
Rodjanar Danchanchai	Demonstration School, Khon Kaen University
Ketsaraporn Suanse	Demonstration School, Khon Kaen University
Arkom Eungpuang	Demonstration School, Khon Kaen University
Viroj Thumnong	Demonstration School, Khon Kaen University
Paitoon Narakhon	Demonstration School, Khon Kaen University
Ruethairat Thomsutha	Demonstration School, Khon Kaen University
Pramote Saeuhan	Demonstration School, Khon Kaen University
Meena Wongsalee	Demonstration School, Khon Kaen University
Kamontip Sombattheera	Chumchon Ban Chonnabot School
Buakaew Hirigokul	Chumchon Ban Chonnabot School
Yupapak Sadoi	Chumchon Ban Chonnabot School
Ukrit Chamnanprei	Chumchon Ban Chonnabot School
Somjai Maneevong	Kookham Pittayasan School
Cholathorn Nonting	Kookham Pittayasan School
Lamoon Sanboot	Kookham Pittayasan School
Jansee Laopadang	Kookham Pittayasan School
Auttasart Nimitpun	Suan Dusit Rajabhat University
Sukanya Hajihalah	Srinakarinwirot University
Jongkol Tumsoun	Srinakarinwirot University
Songchai Ugsonkid	Srinakarinwirot University
Jensamut Saengpun	Chaing Mai University
Yanin Kongthip	Srinakharinwirot University
Paweena Prawing	Lampang Rajabhat University
Wasukree Jaijan	Rajamangala University of Technology Suvanrnabhumi
Wipaporn Suttiamporn	Center for Research in Mathematics Education
Samphan Jaktinkhajornklai	Khon Kaen University
Somkuan Srichompoo	Center for Research in Mathematics Education
Narumon Changsri	Center for Research in Mathematics Education

NAME	AFFILIATION
Chalayut Crutmuang	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Kesorn Tongsan	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Prawit Wankhom	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Sutsiri Sirikun	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Julamas Sornjansukot	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Pakorn Khanson	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Ketsinee Ponlaboon	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Saman Eakpim	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Panadda Wongjanta	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Nattapong Chaladyam	Graduate Student, Doctor of Education Program in Curriculum and Instruction, Khon Kaen University
Kreangsak Srisombat	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Kawveing Namnapon	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Wichit Kammantakun	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Noppadon Poonsawat	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Kaisit Plerin	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Pinyo Manoosin	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Sirikul Namsiri	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Saisamorn Sakkhamduang	Graduate Student, Doctor of Philosophy Program in Education, Khon Kaen University
Samrit Kangpeng	Graduate Student, Doctor of Philosophy Program in Education, Khonkaen University

NAME	AFFILIATION
Prakaikham Thesarin	Center for Research in Mathematics Education
Thanya Kadroon	Center for Research in Mathematics Education
Kawissara Sansanoh	Center for Research in Mathematics Education
Kruewan Waisang	Mahawichanukool School
Kittipakorn Omthuan	Center for Research in Mathematics Education
Pimlak Moonpo	Center for Research in Mathematics Education
Suwarnnee Plianram	Center for Research in Mathematics Education
Suttharat Boonlerts	Center for Research in Mathematics Education
Rungtiwa Kannakarn	Koo Kaew Wittaya School
Souksomphone Anothay	Muttayomsomboon Sawannaket School, Laos
Thongkhao Sengsoulichanh	Muttayomsuksapakjang School, Laos
Kadumpee Kaseecha	Thamafaiwan School
Katanyuta Bangto	Center for Research in Mathematics Education
Kridsana Doungpila	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Kanjana Wetbunpot	Center for Research in Mathematics Education
Kittisak Jai-on	Center for Research in Mathematics Education
Kessuda Nawglang	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Kasem Premprayoon	Center for Research in Mathematics Education
Thummanat Thobumrung	Banpakkhamphu School
Nittaya Wongsaen	Tessabanmuangpathoomthanee School
Benjawan Chaiplad	Center for Research in Mathematics Education
Preecha Pimkaew	Bansapanyao School
Vayukon Robsouk	Khangkhrowittaya School
Jarinee Imdoung	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Bounthanh Souttaphong	Souphanouvong University
Karnda Chalardlon	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Jaruwan Pakang	Banmaipittayakhom School
Julaluk Jai-on	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Charoen Rakakaew	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Chompoo Srisun	Sikhoraphum Industrial and Community Education College Surin Province
Chatchawan Nampreeda	Nachansuksa School
Tidarat Pojchanataree	Suntiwittayasan School
Prapawadee Pairam	Tessaban 3 (Ban Nongmaung) School
Preechakorn Phachana	Banhunwittaya School
Rungnapa Arayathamsophon	Chumpaesuksa School
Wattana Panjarukse	Rajamangala University of Technology Isan Sakon Nakhon Campus
Siriporn Srichantha	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Supawan Koatthan	Khaoraisuksa School
Suntharee Sawongnam	Graduate Student, Master Program in

NAME	AFFILIATION
	Mathematics Education, Khon Kaen University
Aroonsri Yingyuen	Chumpaesuksa School
Achariya Pongpisanu	Nong Muad Ae Wittaya School
Krit Rattanarat	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Tussawan Lengtamdee	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Phichaown Lng-anurak	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Rungtiwa Wongsorn	Prasatwitthayakhom School
Wantana Joomjan	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Wipaporn Sangsawang	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Sawanee Jamreanvong	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Sudatip Hancherngchai	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Surachai Sukharee	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Patipahn Yodkhuntod	Graduate Student, Master Program in Mathematics Education, Khon Kaen University
Xaysy Linphitham	National University of Laos
Chutima Khamthanee	Chuenchompittayakhan School
Nittaya Udompon	Nampongsuksa School
Phatitta Wierachai	Thakhanthowittayakhan School
Mongkhol Prasertsang	Demonstration School, Khon Kaen University
Saisunee Na-udom	Waengnoisuksa School
Supannika Chanchompoo	Khon Kaen Vithes Suksa Bilingual School

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Chair	Suladda Loipha	Khon Kaen University
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Editor	Maitree Inprasitha	Khon Kaen University